Dirac-Majorana Neutrino Type Oscillation[1] (and Second Leptogenesis[2]) Induced by a Wave Dark Matter

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Based on Phys. Rev. D. **108**, 095028 (2023) [arXiv: 2305.16900][1], [arXiv: 2311.16672][2]

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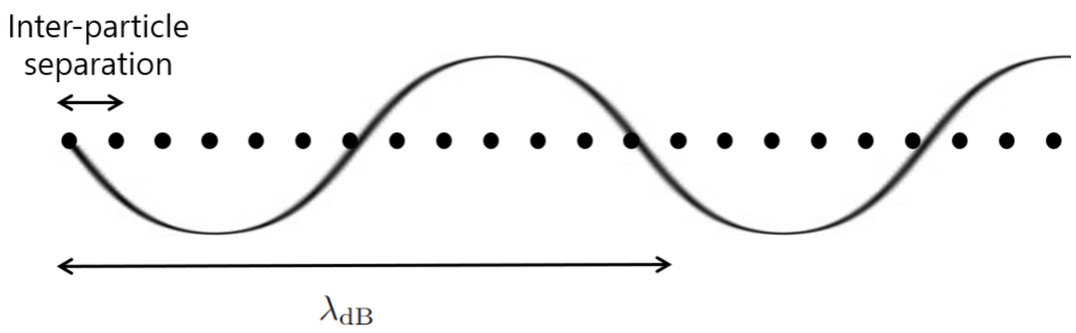
Wave dark matter

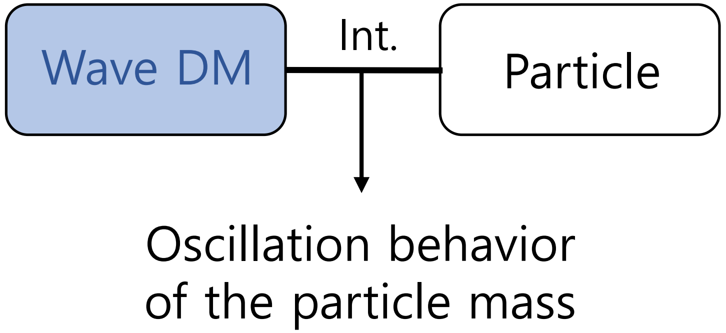
$\lambda_{\rm dB} = \dfrac{2\pi}{mv}$ > Inter-particle separation if $m < 30~\mathrm{eV}$

(Ultralight mass scale)



Equation of Motion: $\ddot{\phi} + \cancel{3H\dot{\phi}} + m_\phi^2 \phi = 0$

Solution: $\phi(t) = \dfrac{\sqrt{2\rho_\phi}}{m_\phi} \cos m_\phi t$



$\mathcal{L} \supset -g\phi\bar{\nu}\nu + h.c.$ → $m_\nu(t) = g\dfrac{\sqrt{2\rho_\phi}}{m_\phi} \cos m_\phi t$

Neutrino mass variation

[A. Berlin (2016), Y. Zhao (2017),
G. Krnjaic et al (2017), V. Brdar et al (2017) …]

$$\mathcal{L} = \overbrace{\boxed{- m_D \bar{\nu}_D \nu_D}}^{\text{Dirac mass}} \overbrace{\boxed{- \frac{1}{2} m_R \bar{\nu}_M \nu_M}}^{\text{Majorana mass}}$$

$$\nu_D = \nu_L + \nu_R$$

$$\nu_M = \nu_R + \nu_R^c$$

Majorana type $(m_D \ll m_R)$

$$\mathcal{L} = - m_D \bar{\nu}_D \nu_D - \frac{1}{2} m_R \; \bar{\nu}_M \nu_M$$

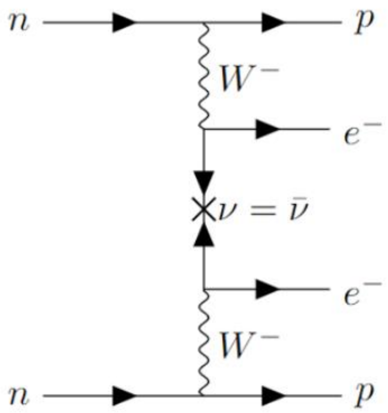
Seesaw mechanism

(for small m_ν)

$$m_\nu = - \frac{m_D^2}{m_R}$$

Neutrinoless double beta decay

(as LNV process)



Quasi-Dirac type $(m_D \gg m_R)$

$$\mathcal{L} = - m_D \bar{\nu}_D \nu_D - \frac{1}{2} m_R \; \bar{\nu}_M \nu_M$$

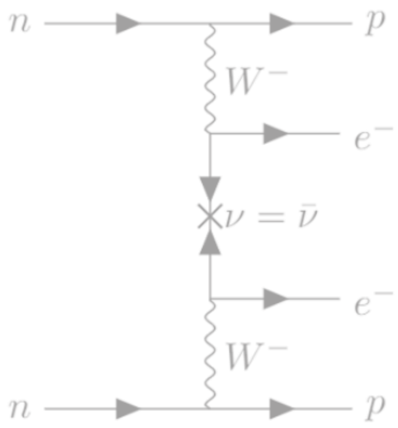
Seesaw mechanism

Seesaw is broken

$$m_\nu = - \frac{m_D^2}{m_R}$$

Neutrinoless double beta decay

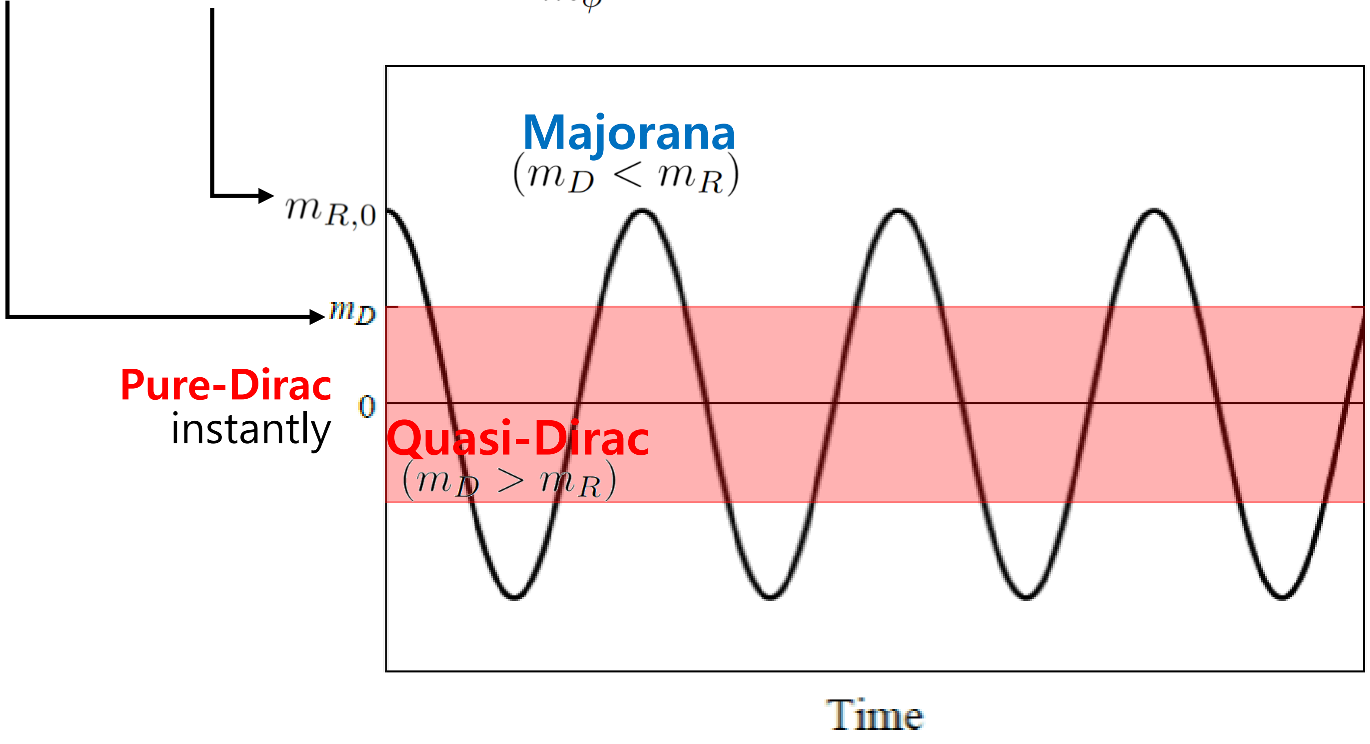
LNV is suppressed

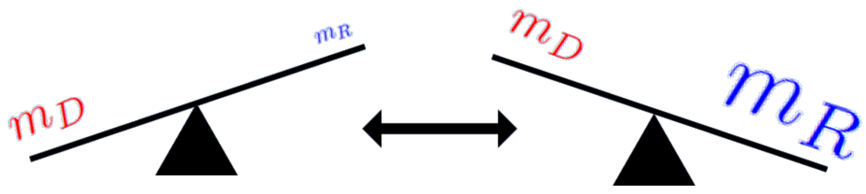


$$\mathcal{L} = -yHLN - g\,\phi NN + h.c.$$

$$m_D = \frac{yv}{\sqrt{2}} \qquad m_R(t) = m_{R,0}\cos m_\phi t$$

(fixed) where $m_{R,0} = g\frac{\sqrt{2\rho_\phi}}{m_\phi}$

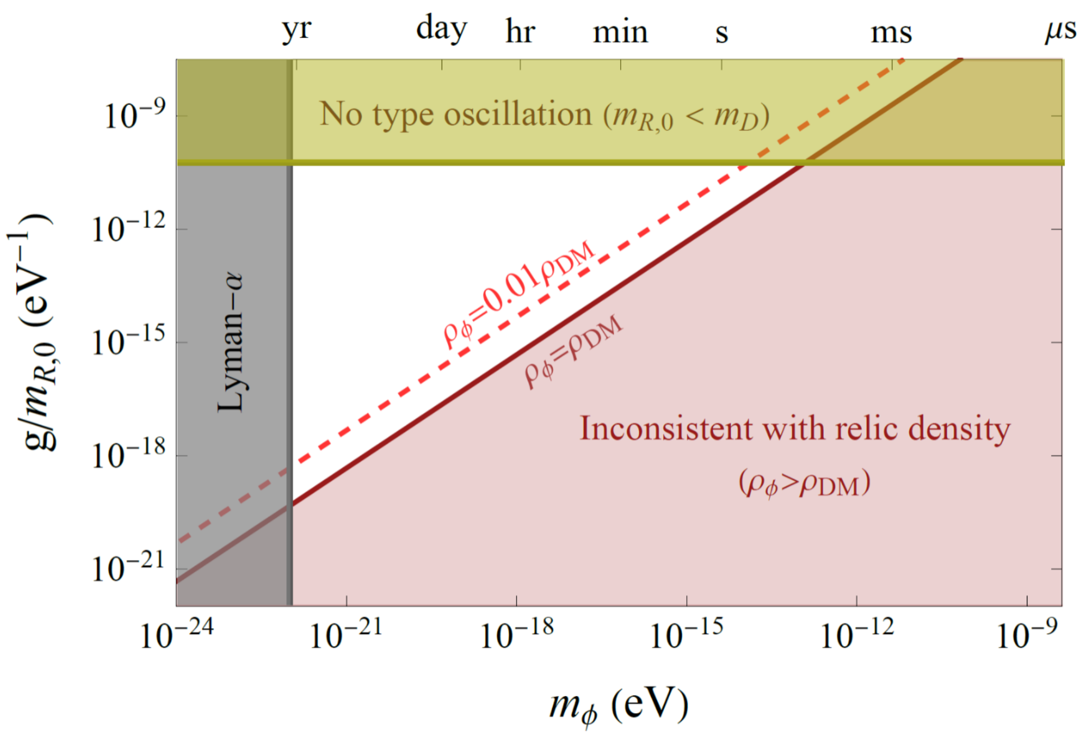


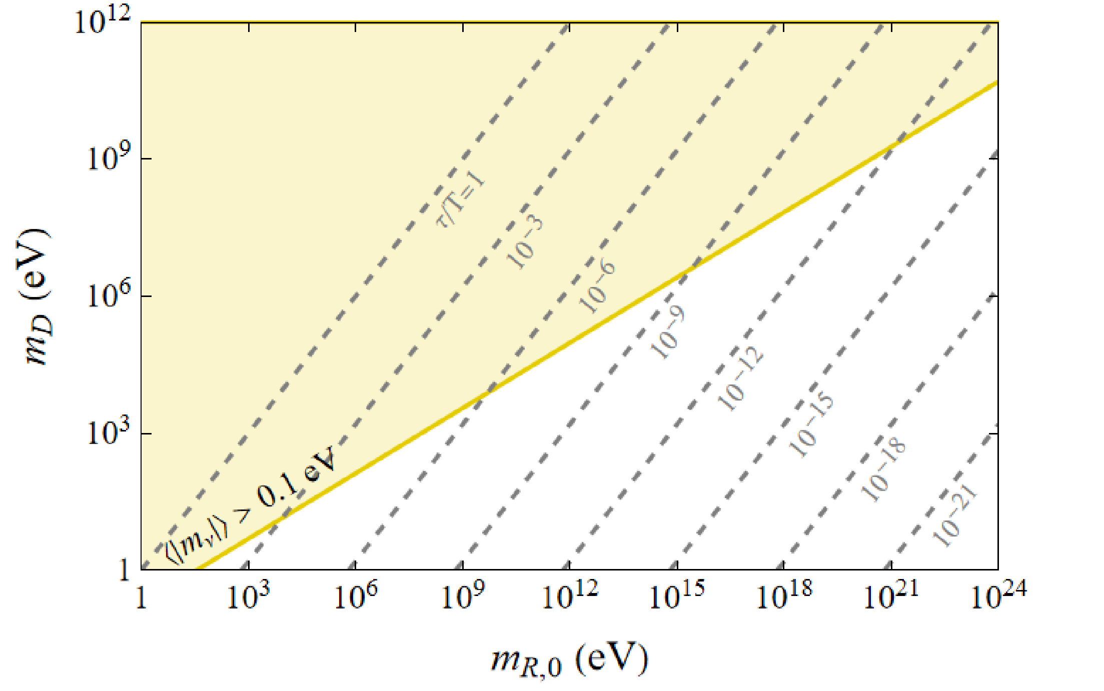


Quasi-Dirac type Majorana type

Wave Dark Matter

"Dirac-Majorana neutrino type oscillation"





(1) Wave DM mass range $10^{-22}\ \mathrm{eV} < m_\phi < 30\ \mathrm{eV}$

(2) Constraint on DM relic density

$$m_{R,0} \simeq 10^{19}\ \mathrm{eV}\left(\frac{g}{1}\right)\left(\frac{\rho_\phi}{\rho_{\mathrm{DM}}}\right)^{1/2}\left(\frac{10^{-22}\ \mathrm{eV}}{m_\phi}\right)$$

(3) Disfavored region for type oscillation

$m_{R,0} < m_D$ with $g < 1$, $m_D \simeq 246\ \mathrm{GeV}$

Time average

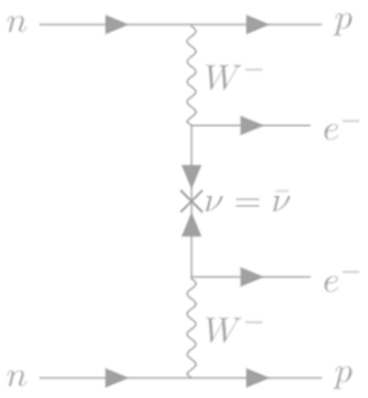
Mass bound
from CMB/^3H decay

$$\langle|m_\nu|\rangle = \frac{1}{T}\int_0^T |m_\nu(t)|dt < 0.1\ \mathrm{eV}$$

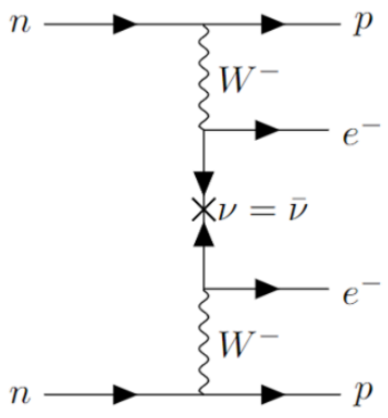
→ Short Quasi-Dirac moment for small neutrino mass

(Also for neutrino flavor oscillation)

Neutrino type oscillation



LNV

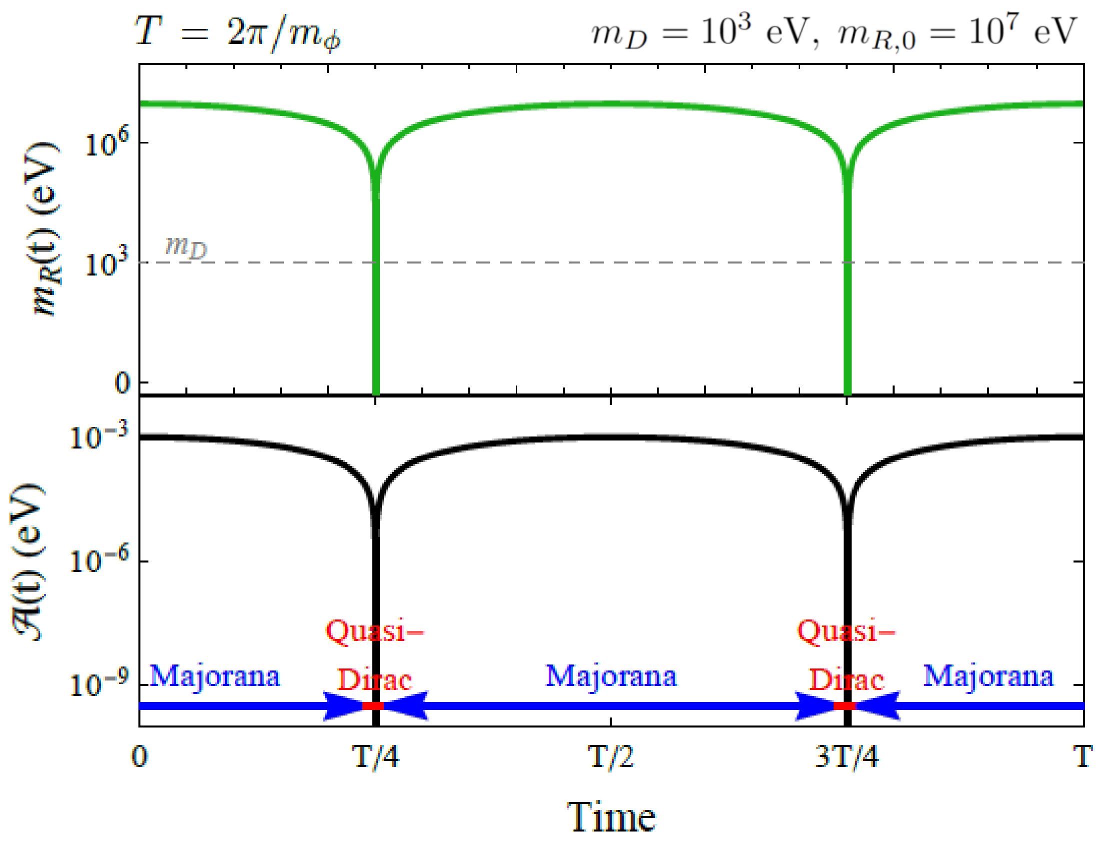


LNV

Quasi-Dirac type ⟷ Majorana type

$$\Gamma_{0\nu\beta\beta} \propto \mathcal{A}^2 = \left| \sum_i U_{ei}^2 m_i \mathcal{M}(m_i) \right|^2$$

Turn-on/off behavior of 0νββ decay



Leptogenesis (LG)

Majorana mass term

$$\mathcal{L} = -yHLN \boxed{- MNN} + h.c.$$

CP asymmetry

$$\epsilon_i = \frac{\Gamma(N_i \to \ell\Phi) - \Gamma(N_i \to \bar{\ell}\Phi^\dagger)}{\Gamma(N_i \to \ell\Phi) + \Gamma(N_i \to \bar{\ell}\Phi^\dagger)}$$

Heavy Majorana neutrino decay

$N_i \to \ell\Phi$, $N_i \to \bar{\ell}\Phi^\dagger$

(N_i is mass eigenstate)

↓

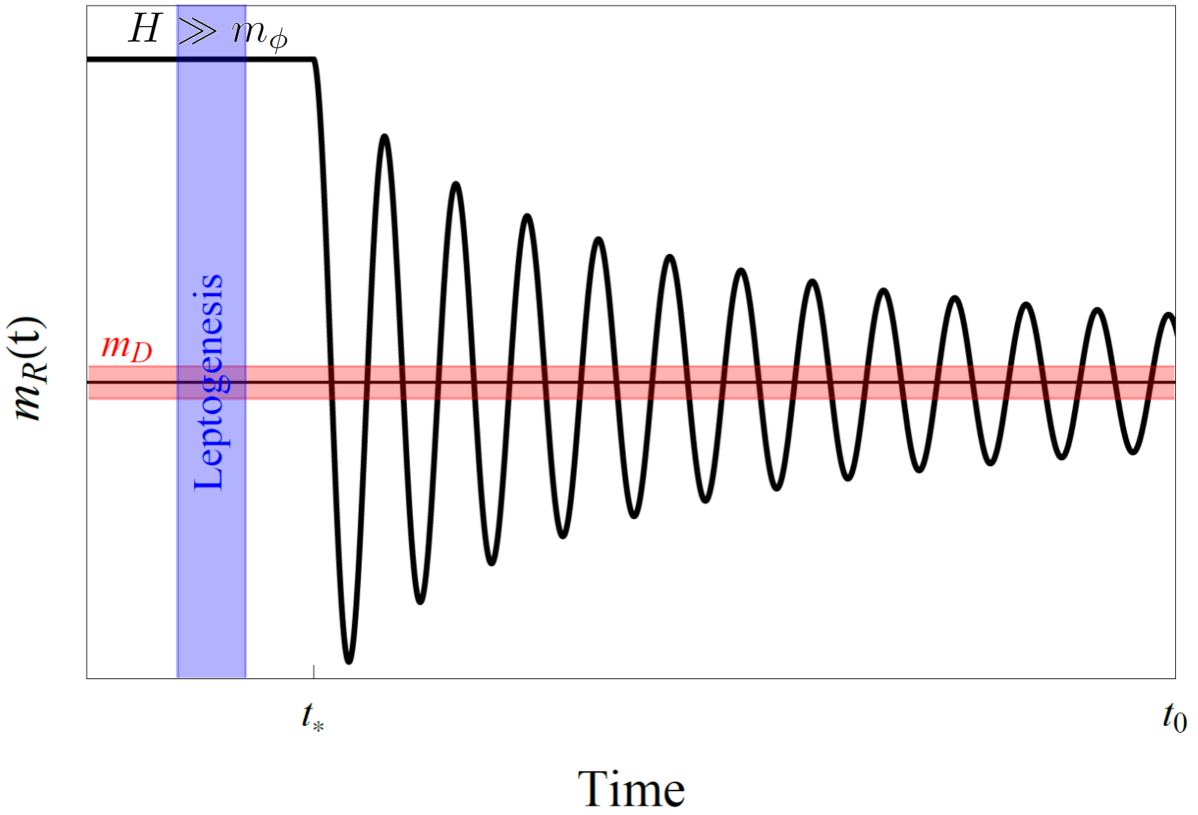
Lepton asymmetry

↓ Sphaleron process
(*B*-*L* is conserved)

Baryon asymmetry of the universe (BAU)

$$\eta_B \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} \simeq (6.14 \pm 0.25) \times 10^{-10}$$

$$\ddot{\phi} + \boxed{3H\dot{\phi}} + m_\phi^2 \phi = 0$$



$$m_R \xrightarrow{m_R = g\frac{\sqrt{2\rho_\phi}}{m_\phi} \propto a^{-3/2}} m_R$$

Early universe

Present

LG

Type Osc.

Leptogenesis with the **Neutrino mass variation**

Production rate of heavy neutrino

$$\Gamma_{\rm prod} \propto e^{-M/T} \quad \text{with} \quad M(T) \propto T^{3/2}$$

$T > M(T)$ Equilibrium-in

$T < M(T)$ Decoupling

M(T) has different slope with temperature *T*.

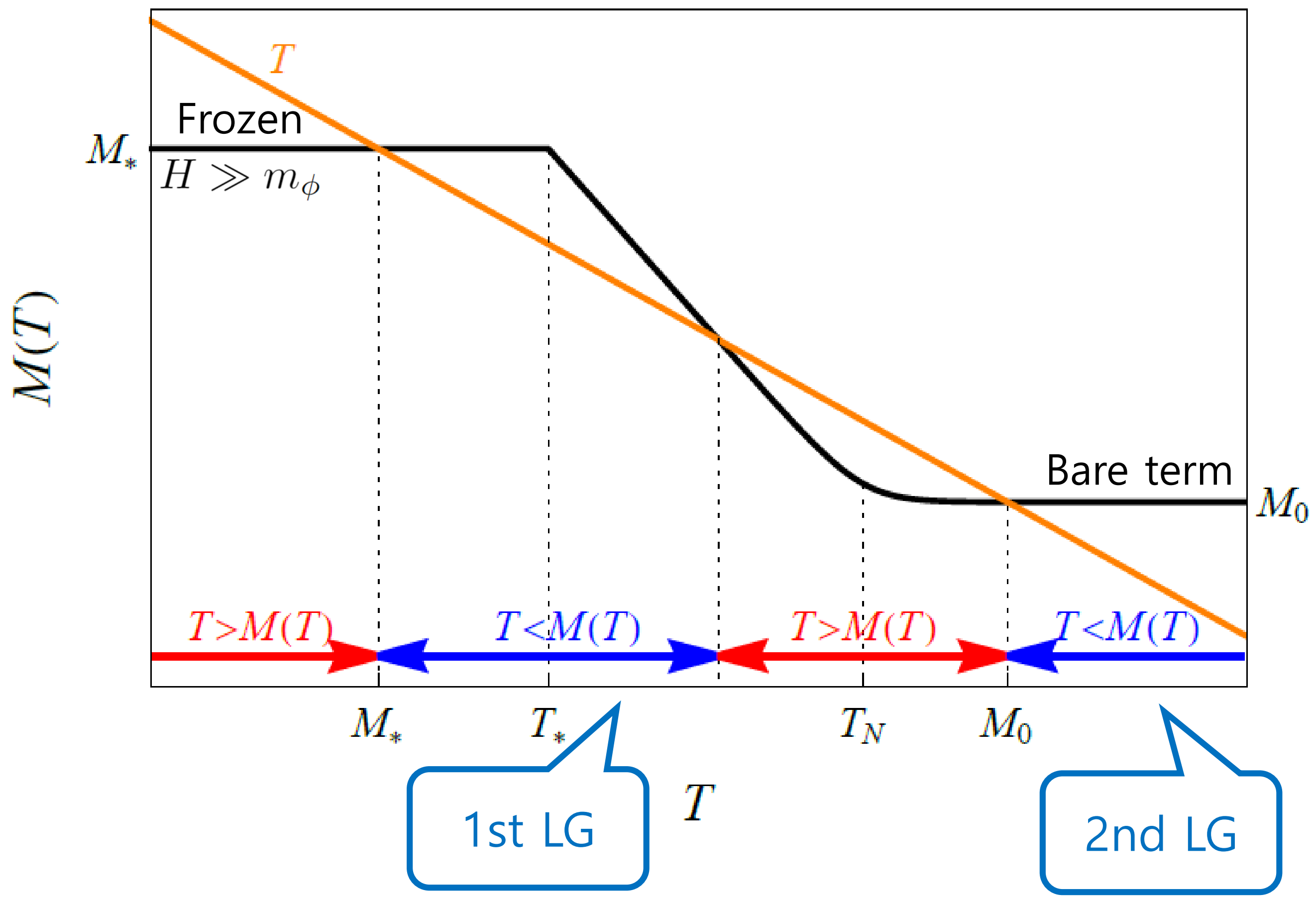
→ Hierarchy between *M* and *T* can be changed.

($M \simeq m_R$ is mass eigenvalue)

Consider $\mathcal{L} \supset (M_0 + g\phi)\overline{N^c}N + h.c.$

Bare mass — Dominant when $T_N > T$ (present-time)

$M(T)$ — Dominant in the early universe



"Second Leptogenesis"

Lepton asymmetry

$$\eta_L \equiv \frac{n_L - n_{\bar{L}}}{n_\gamma} \simeq (7.5^{+4.5}_{-3.0}) \times 10^{-2}$$

(reported by EMPRESS

with $^4\mathrm{He}$ abundance observation)

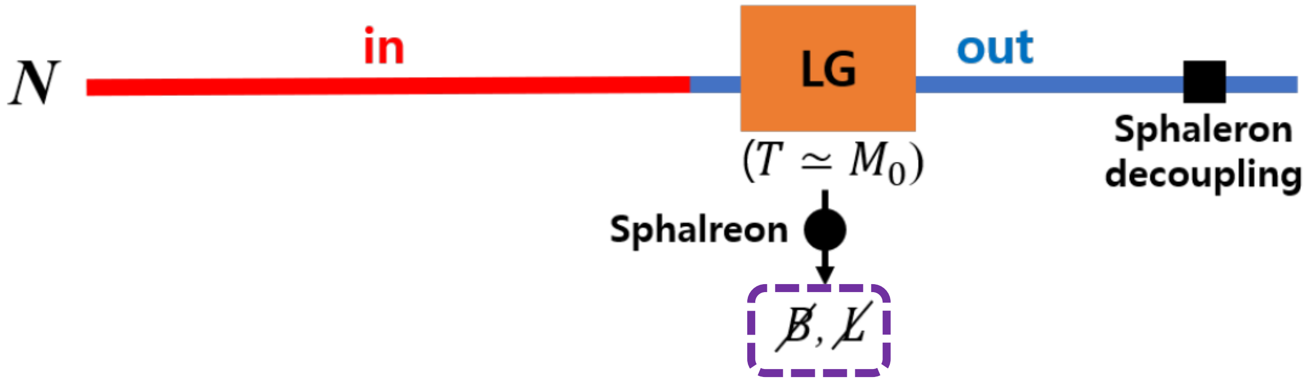
[A. Matsumoto et al (2022)]

Large discrepancy!

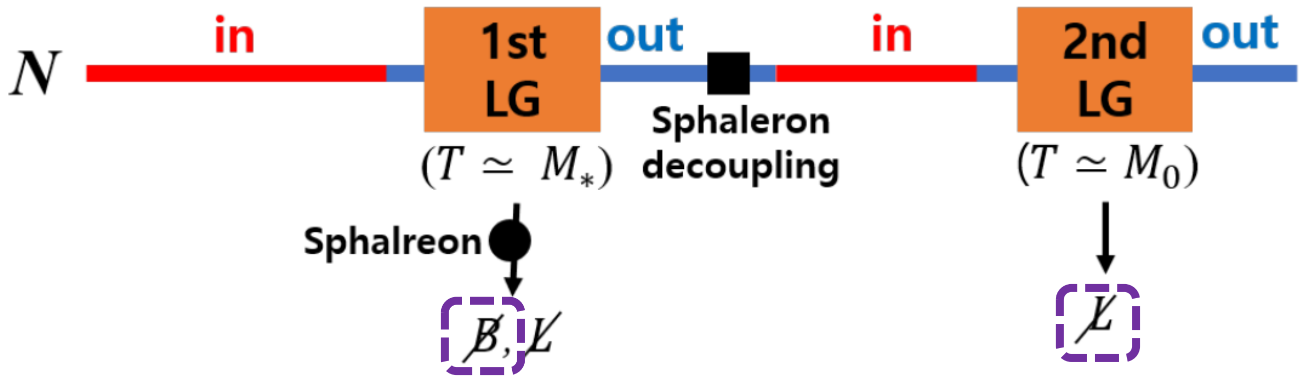
Baryon asymmetry

$$\eta_B \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} \simeq (6.14 \pm 0.25) \times 10^{-10}$$

(a) **Standard case without mass variation**



(b) **Twofold LG with mass variation**



→ If sphalereon decoupling occurs between 1st and 2nd LG,
the baryon asymmetry and lepton asymmetry can be different.

Boltzmann equation (density matrix form)

Plug the mass varying effect

$$\frac{\mathrm{d}N_{N_i}}{\mathrm{d}z} = -\boxed{D_i}\left(N_{N_i} - \boxed{N_{N_i}^{\mathrm{eq}}}\right)$$

$$\frac{\mathrm{d}N_{\alpha\beta}}{\mathrm{d}z} = \sum_i \left[\epsilon_{i,\alpha\beta}\boxed{D_i}\left(N_{N_i} - \boxed{N_{N_i}^{\mathrm{eq}}}\right) - \frac{1}{2}\boxed{W_i}\{P_i, N\}_{\alpha\beta}\right] - \frac{\Gamma_\tau}{Hz}[I_\tau,[I_\tau,N]]_{\alpha\beta} - \frac{\Gamma_\mu}{Hz}[I_\mu,[I_\mu,N]]_{\alpha\beta}$$

See [A. Granelli et al (2021)] for detailed formula

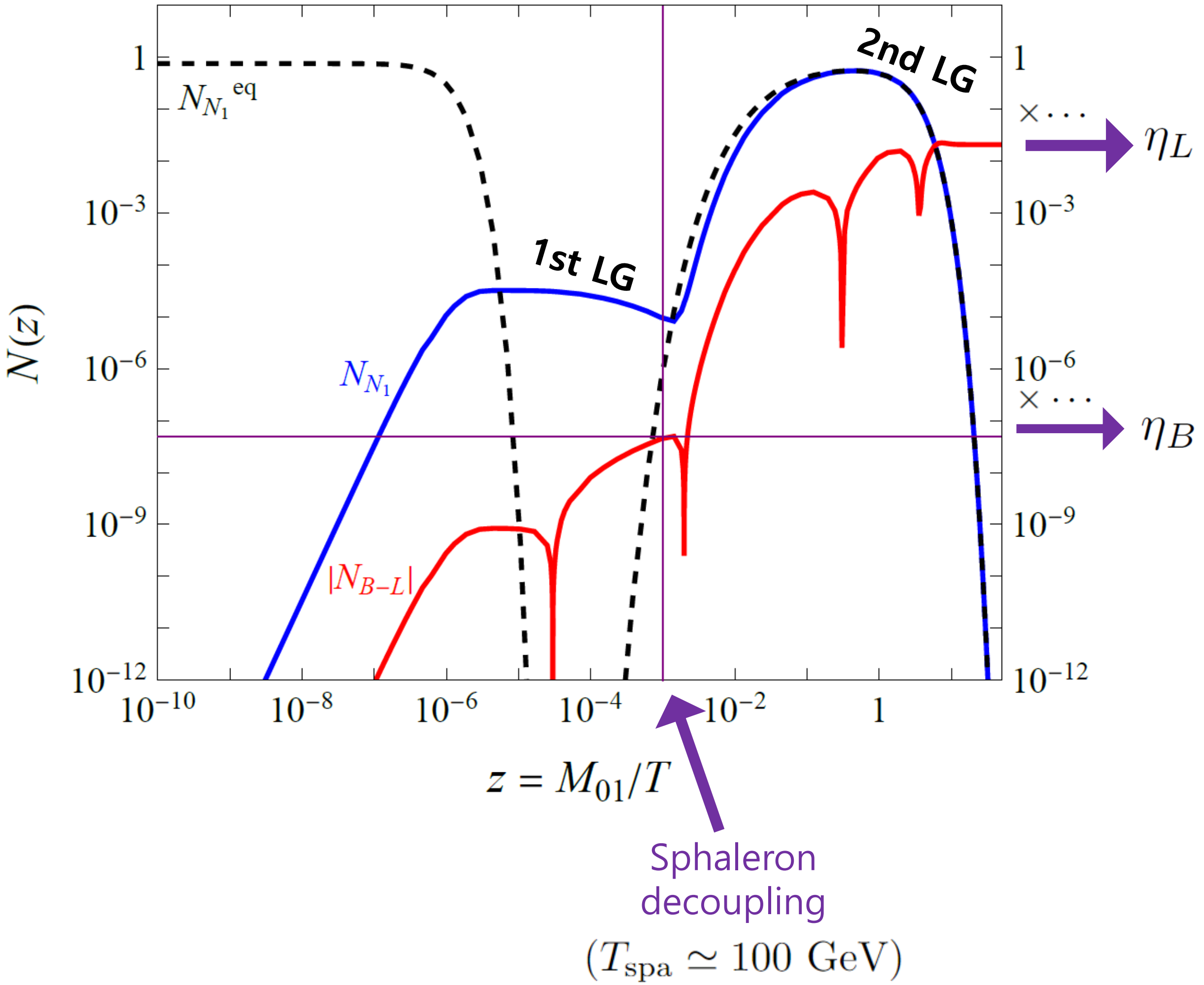
$$\eta_L = \frac{1}{f'}N_{B-L} \qquad f' = 176/43$$

$$\eta_B = \frac{a_{\mathrm{spa}}}{f}N_{B-L} \qquad a_{\mathrm{spa}} = 28/79,\ f = 1232/43$$

Benchmark point

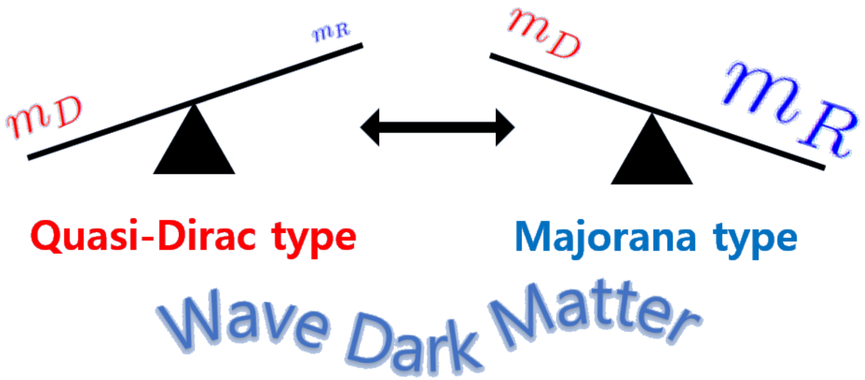
$m_\phi = 10^{-2}$ eV, $M_0 = 0.1$ GeV, $M_* = 2.4\times10^5$ GeV (for N_1)

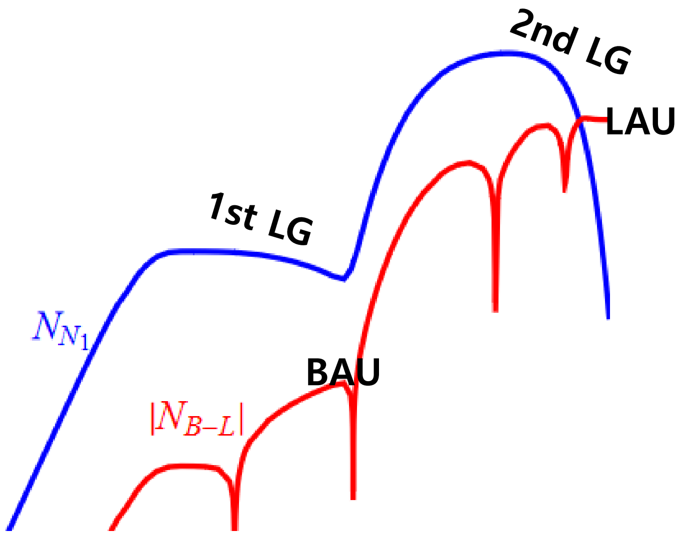
$\cdots$ See our paper for detail



Summary

- The **wave dark matter** can be source of the mass variation of the particle.
- If the ratio of the Dirac and Majorana mass term changes in time, the neutrino type may oscillate between quasi-Dirac type and Majorana type, so-called **Dirac-Majorana neutrino type oscillation**.
- The neutrino type oscillation predicts the modulation of 0νββ decay.
- The mass scale evolves over cosmic history that gives link between early universe and present-time.
- The neutrino mass variation scenario provides **second leptogenesis**, which can be source of the large discrepancy between the lepton asymmetry and baryon asymmetry of the universe.





Backup Slides

Dirac/Majorana spinors formalism

$$\mathcal{L} = -yHLN - MNN + h.c.$$

Dirac neutrino $\nu_D = \nu_L + \nu_R$

Majorana neutrino $\nu_M = \nu_R + \nu_R^c$

Dirac mass term Majorana mass term

$$\mathcal{L} = \boxed{-m_D \bar{\nu}_D \nu_D} \boxed{-\frac{1}{2} m_R \bar{\nu}_M \nu_M}$$

$$= -\frac{1}{2} \begin{pmatrix} \bar{\nu}_L & \bar{\nu}_R^c \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D & m_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix} + h.c.$$

light/heavy mass eigenvalues:

$$m_{l,h} = \frac{1}{2}\left(m_R \mp \sqrt{m_R^2 + 4m_D^2}\right)$$

light/heavy mass eigenstates:

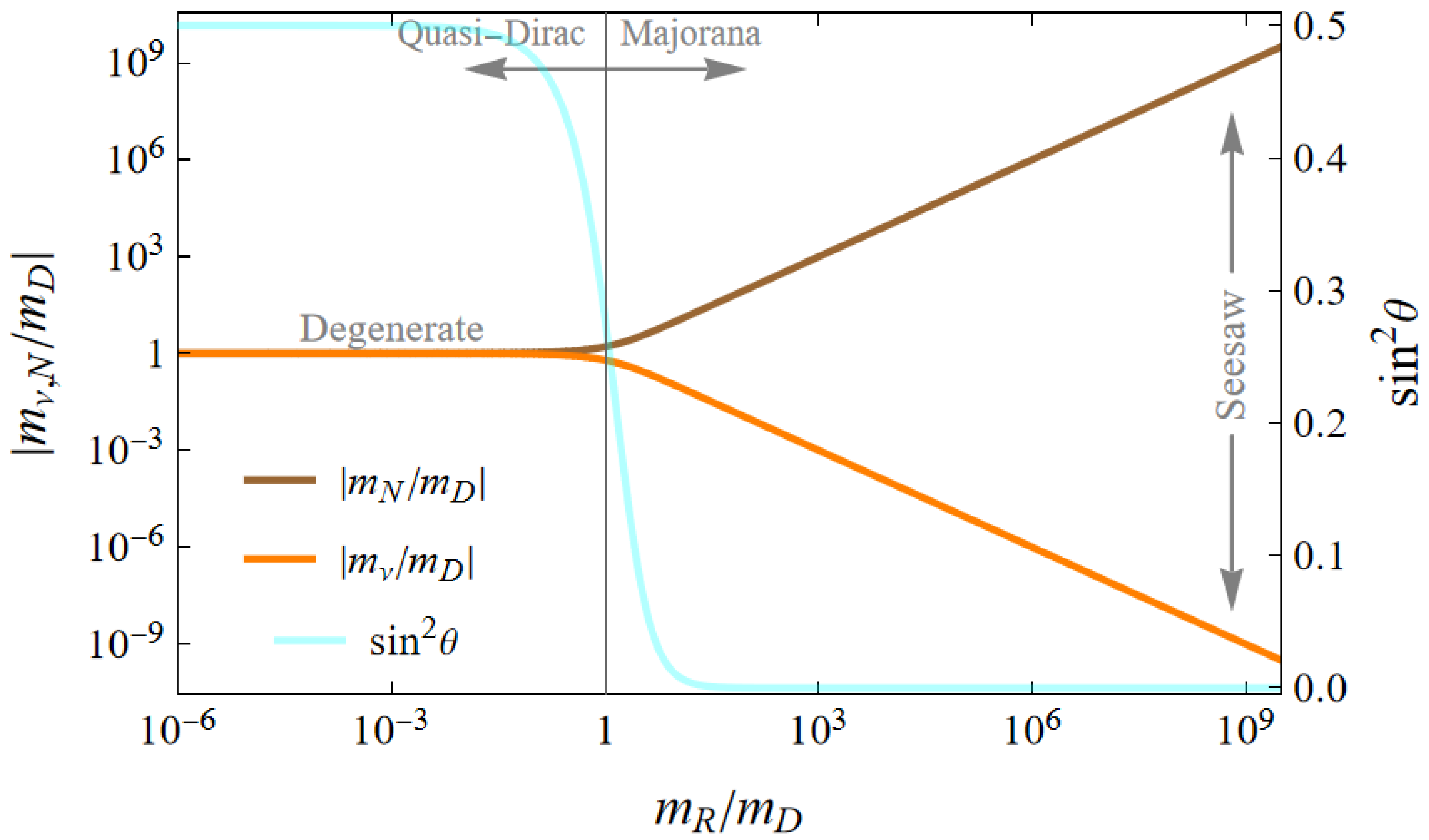
$$\begin{pmatrix} \nu_l \\ \nu_h \end{pmatrix} = \begin{pmatrix} \cos\theta_{LR} & \sin\theta_{LR} \\ -\sin\theta_{LR} & \cos\theta_{LR} \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix} + h.c.$$

$$\text{where } \sin^2\theta_{LR} = \frac{1}{2}\left(1 - \frac{m_R}{\sqrt{m_R^2 + 4m_D^2}}\right)$$

Quasi-Dirac & Majorana limits

	Pure Dirac case $(m_R = 0)$	Quasi-Dirac limit $(m_D \gg m_R)$	Majorana limit $(m_D \ll m_R)$
Mass eigenvalues	$\lvert m_{l,h}\rvert = m_D$	$\lvert m_{l,h}\rvert \simeq m_D(1 \mp 2\delta)$ where $\delta = m_R/4m_D$	$\boxed{m_l \simeq -\frac{m_D^2}{m_R}}, \; m_h \simeq m_R$ Seesaw mechanism
Mixing angle	$\sin^2\theta_{LR} = \frac{1}{2}$	$\sin^2\theta_{LR} \simeq \left(\frac{1-\delta}{\sqrt{2}}\right)^2$	$\sin^2\theta_{LR} \simeq \left(\frac{m_D}{m_R}\right)$
Mass eigenstates	$\nu_{l,h} = \frac{1}{\sqrt{2}}\Big((\nu_L + \nu_L^c) \pm (\nu_R + \nu_R^c)\Big)$	$\nu_{l,h} \simeq \pm\frac{1 \pm \delta}{\sqrt{2}}(\nu_L + \nu_L^c) + \frac{1 \mp \delta}{\sqrt{2}}(\nu_R + \nu_R^c)$	$\nu_l \simeq \nu_L + \nu_L^c, \; \nu_h \simeq \nu_R + \nu_R^c$
Mass term	$\frac{1}{2}(m_l\bar{\nu}_l\nu_l + m_h\bar{\nu}_h\nu_h) = m_D\bar{\nu}_D\nu_D$	$\frac{1}{2}(m_l\bar{\nu}_l\nu_l + m_h\bar{\nu}_h\nu_h) \simeq m_D\bar{\nu}_D\nu_D + \mathcal{O}(\delta)$	$\frac{1}{2}(m_l\bar{\nu}_l\nu_l + m_h\bar{\nu}_h\nu_h)$

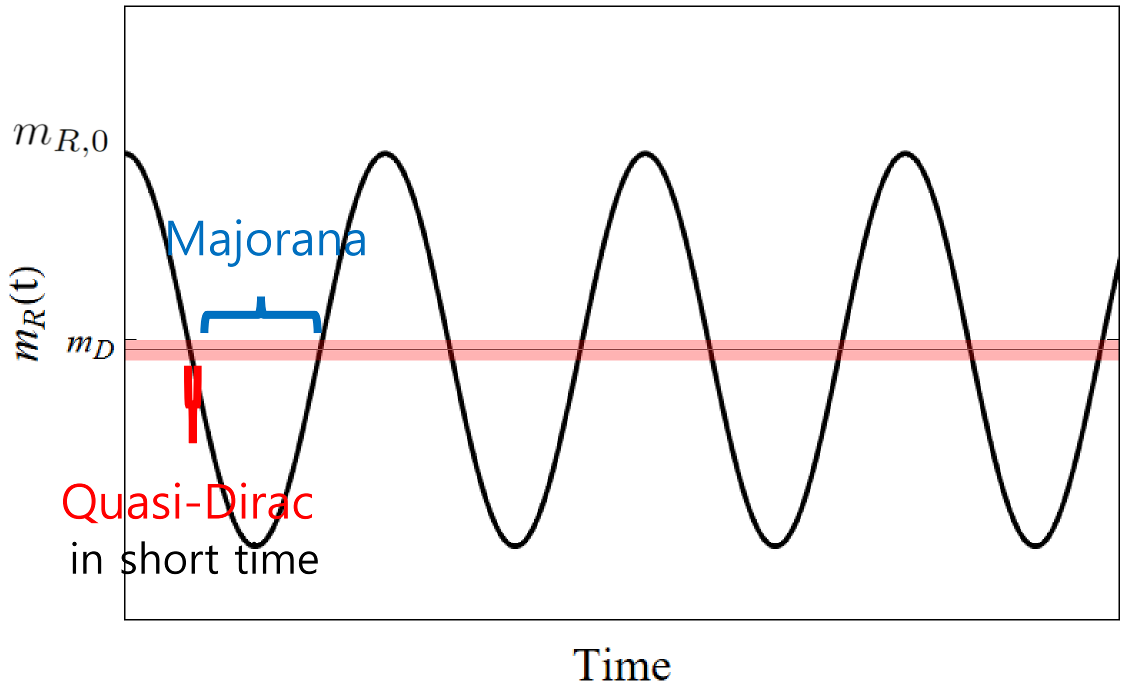
Quasi-Dirac & Majorana limits



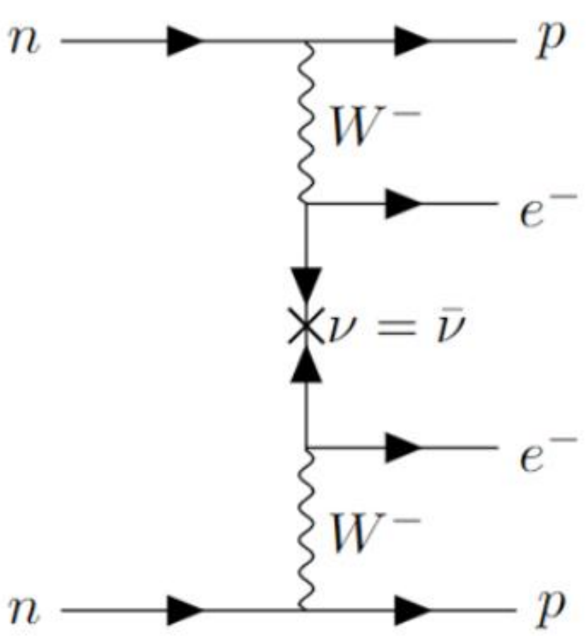
Quasi-Dirac time ratio τ/T

$$\frac{\tau}{T} = 1 \qquad \text{for } m_D > m_{R,0}$$

$$\frac{m_D}{m_{R,0}} = \sin\left(\frac{\pi}{2}\frac{\tau}{T}\right) \qquad \text{for } m_D < m_{R,0}$$



Neutrinoless double beta decay (0νββ): only for massive Majorana neutrinos



$$\Gamma_{0\nu\beta\beta} \propto \mathcal{A}^2$$

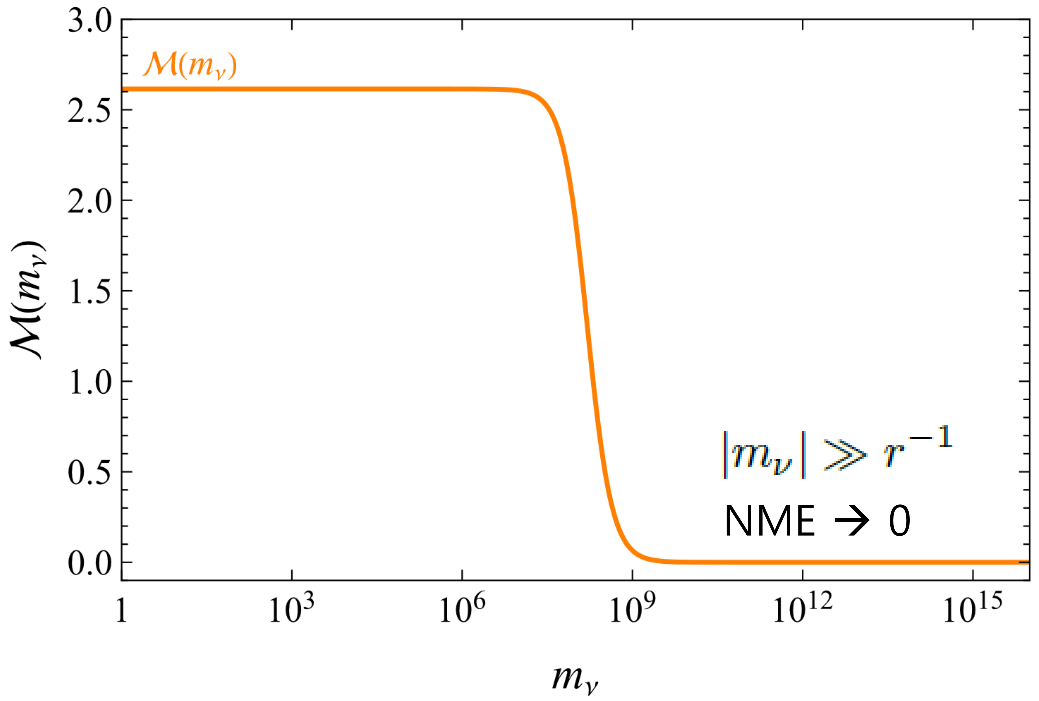
$$\mathcal{A} = \left| \sum_i \underline{U_{ei}^2} m_i \underline{\mathcal{M}(m_i)} \right|$$

Extended PMNS matrix

Nuclear matrix element (NME)

$$\mathcal{M}(m_\nu) = m_p m_e \frac{|\mathcal{M}_N^{0\nu}|}{\langle p \rangle^2 + m_\nu^2}$$

$$\text{where } \langle p \rangle^2 = m_p m_e \left| \frac{\mathcal{M}_N^{0\nu}}{\mathcal{M}_\nu^{0\nu}} \right|$$



r : inter-nucleon distance (fm)

0νββ decay with Type oscillation

Single-flavor case:

$$\mathcal{A} = \left| m_l \cos^2\theta_{LR}\mathcal{M}(m_l) + m_h \sin^2\theta_{LR}\mathcal{M}(m_h) \right|$$

$|m_\nu| \simeq |m_N|$, $\cos^2\theta \simeq \sin^2\theta$ in quasi-Dirac limit

→ $\mathcal{A}$ is strongly suppressed

Majorana limit in the most of the time:

$\sin^2\theta_{LR} \simeq 0$, $\mathcal{M}(m_h) \simeq 0$ → 0νββ bound is reduced into $\langle |m_l| \rangle$ bound

Three-flavors case: $\mathcal{A} = \left| \sum_i U_{ei}^2 m_i \mathcal{M}(m_i) \right|$

Mixing matrix element

$$U_{e1} = \cos\theta_{13}\cos\theta_{12}\cos\theta_{LR}^{11}$$
$$U_{e2} = \cos\theta_{13}\sin\theta_{12}\cos\theta_{LR}^{22}$$
$$U_{e3} = \sin\theta_{13}\cos\theta_{LR}^{33}$$
$$U_{e4} = \cos\theta_{13}\cos\theta_{12}\sin\theta_{LR}^{11}$$
$$U_{e5} = \cos\theta_{13}\sin\theta_{12}\sin\theta_{LR}^{22}$$
$$U_{e6} = \sin\theta_{13}\sin\theta_{LR}^{33}$$

Three pairs of light/heavy mass term → $\mathcal{A}$ is suppressed in quasi-Dirac limit

Lepton asymmetry

$$\eta_L \equiv \frac{n_L - n_{\bar{L}}}{n_\gamma} \simeq \sum_l \frac{g_l \pi^2}{12\zeta(3)} \left(\frac{T_l}{T_\gamma}\right)^3 \xi_l$$

FD distribution

$$\xi_e \equiv \frac{\mu_{\nu_e}}{T} = 0.05^{+0.03}_{-0.02}$$

EMPRESS measurement of $^4\mathrm{He}$ abundance

$$\eta_L \simeq \frac{\pi^2 \sum_{i=e,\mu,\tau} \xi_{\nu_i}}{6\zeta(3)} \left(\frac{T_\nu}{T_\gamma}\right)^3 \simeq (7.5^{+4.5}_{-3.0}) \times 10^{-2}$$

$$\xi_{\nu_e} = \xi_{\nu_\mu} = \xi_{\nu_\tau}$$

Flavor universality (due to flavor oscillation)

Charged lepton asymmetry ~ η_B (small) (due to electrically neutral universe)

Spaleron process $(T \geq 100\ \mathrm{GeV})$

SU(2) gauge interaction $\mathcal{L} = \sum_i \bar{\psi}^i_L \gamma^\mu D_\mu \psi^i_L$

$$\psi^i_L = \{q^i_L,\ l^i_L\}$$

$$D_\mu = \partial_\mu - ig\tau^a W^a_\mu$$

Anomaly via triangle diagram $\partial^\mu j^i_\mu = \frac{1}{64\pi^2} F^a_{\mu\nu} \tilde{F}^{\mu\nu a}$ (for Noether current corresponding to global symmetry)

$\Delta N^i_F = \int d^4x j^i_\mu$ is nonzero

During process, $\Delta L_e = \Delta L_\mu = \Delta L_\tau = \Delta B/3$

→ $B - L$ is conserved

Setup for Second LG

$$\mathcal{L} = -\frac{1}{2}(M_{0i} + g_i\phi)\overline{N}_i^c N_i + \text{h.c.}$$

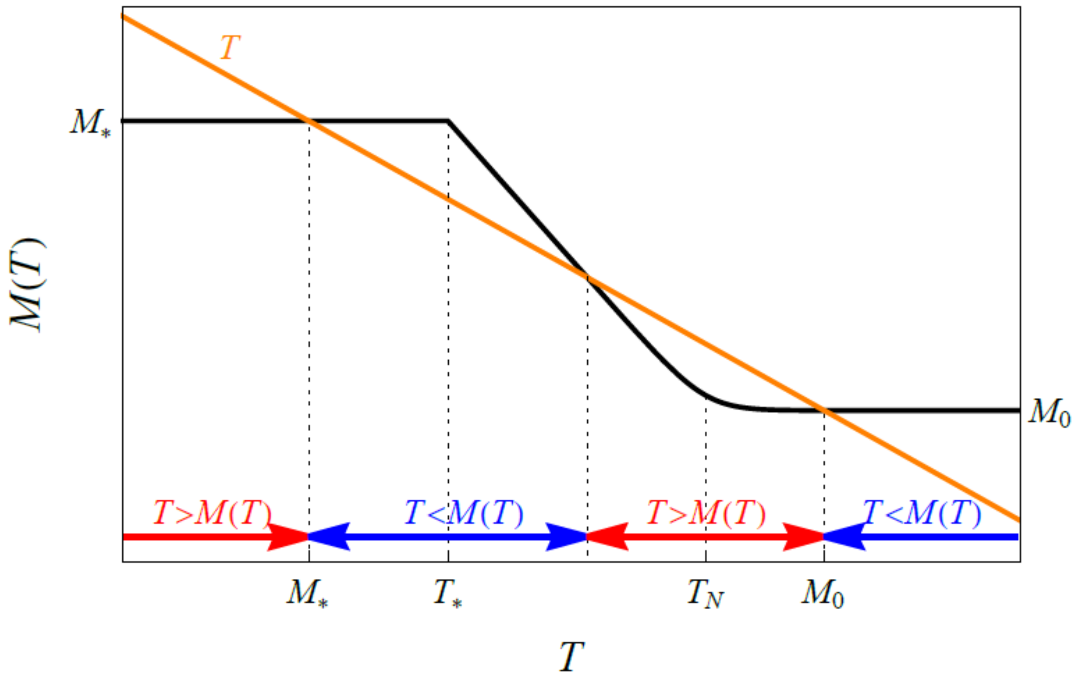
$$M_i(t) = M_{0i} + g_i\phi(t)$$

$$M_i(T) \simeq \begin{cases} M_{*i} & T > T_*, \\ M_{0i} + g_i\frac{\phi_0}{\sqrt{2}}\left(\frac{T}{T_0}\right)^{3/2} & T_* > T > T_{N_i} \\ M_{0i} & T_{N_i} > T, \end{cases}$$

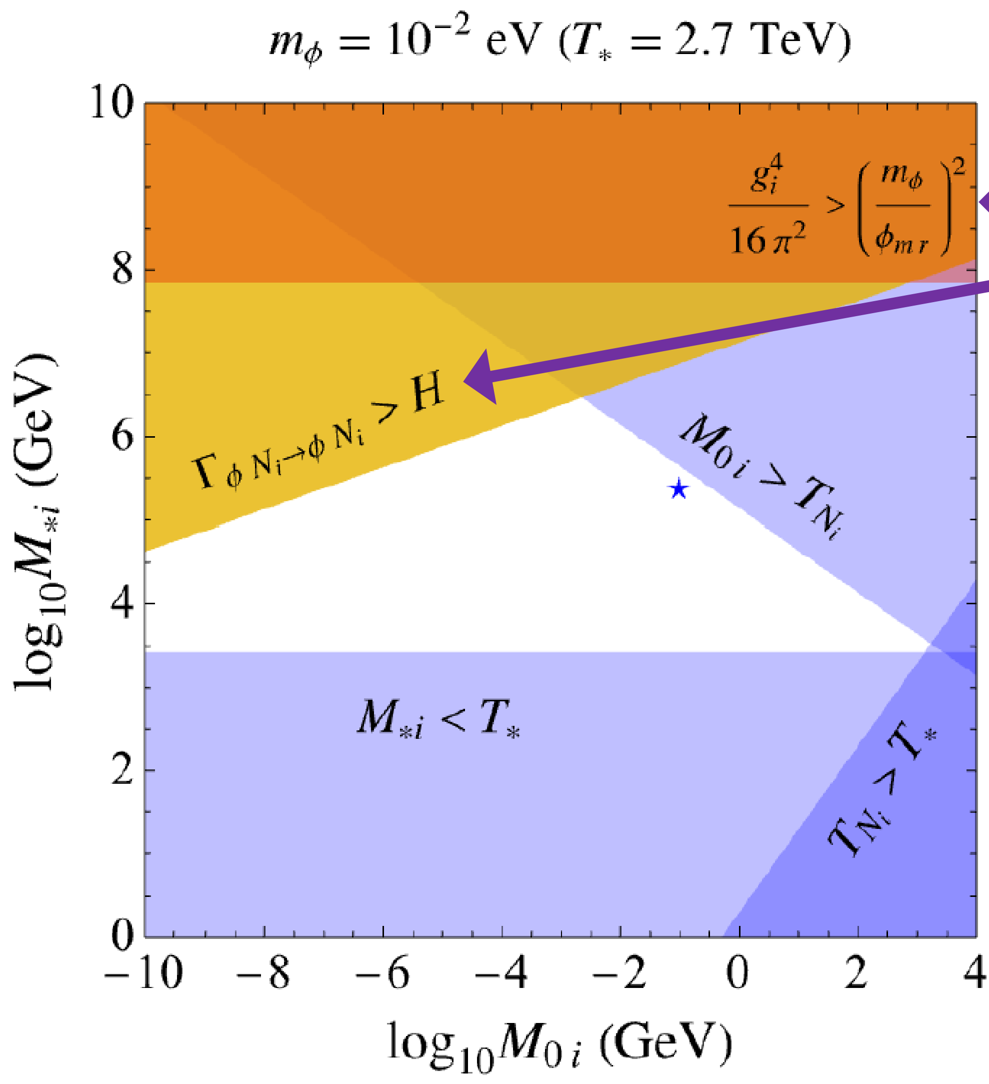
where $M_{*i} \equiv M_{0i} + g_i\phi_0(T_*/T_0)^{3/2}/\sqrt{2}.$

$$T_* \simeq \left(m_\phi M_{\rm Pl}\sqrt{90/(8\pi^3 g_*)}\right)^{1/2}$$

$$M_{0i} = g_i\phi_0(T_{N_i}/T_0)^{3/2}/\sqrt{2}$$



Allowed region for Second LG



To avoid dominant quartic coupling (behaves like radiation-like)

To avoid thermalization of wave DM by scattering [A. Dev et al (2023)]

Boltzmann equation

$$\frac{\mathrm{d}N_{N_1}}{\mathrm{d}z} = -D(N_{N_1} - N_{N_1}^{\mathrm{eq}}),$$
$$\frac{\mathrm{d}N_{B-L}}{\mathrm{d}z} = -\epsilon_1 D(N_{N_1} - N_{N_1}^{\mathrm{eq}}) - W N_{B-L}, \quad \text{where } z = M/T$$

Decay term

Washout term

$$\frac{dN_{N_1}}{dz} > 0 \text{ when } N_{N_1} < N_{N_1}^{\mathrm{eq}}$$
$$\frac{dN_{B-L}}{dz} > 0 \text{ when } N_{N_1} < N_{N_1}^{\mathrm{eq}} \text{ and } W \text{ term is sub-dominant}$$
$$\frac{dN_{B-L}}{dz} > 0 \text{ when } N_{B-L} < 0 \text{ and } W \text{ term is dominant}$$

Boltzmann equation as Density matrix form

$$\frac{\mathrm{d}N_{N_i}}{\mathrm{d}z} = -D_i(N_{N_i} - N_{N_i}^{\mathrm{eq}})$$

$$\frac{\mathrm{d}N_{\alpha\beta}}{\mathrm{d}z} = \sum_i \left[\epsilon_{i,\alpha\beta} D_i(N_{N_i} - N_{N_i}^{\mathrm{eq}}) - \frac{1}{2} W_i \{P_i, N\}_{\alpha\beta}\right] - \frac{\Gamma_\tau}{Hz}[I_\tau, [I_\tau, N]]_{\alpha\beta} - \frac{\Gamma_\mu}{Hz}[I_\mu, [I_\mu, N]]_{\alpha\beta}$$

where $z = M/T \quad i = 1, 2, 3 \quad \alpha, \beta = e, \mu, \tau$

$\epsilon_{i,\alpha\beta}$ is CP asymmetry as density matrix form

D_i is decay term W_i is washout term

P_i is projection operator (constructed by Yukawa matrix)

Γ_τ and Γ_μ are decay rate involving muon and tau for off-diagonal terms (decoherence effect), respectively

$I_\tau = \mathrm{diag}\ (0, 0, 1)$ and $I_\mu = \mathrm{diag}\ (0, 1, 0)$

CP asymmetry & Casas-Ibarra parametrization

$$\epsilon_i \propto \sum_{k\neq i} \frac{\mathrm{Im}(y^\dagger y)^2_{ik}}{(y^\dagger y)_{ii}(y^\dagger y)_{kk}} \frac{(M_k^2 - M_i^2)M_i\Gamma_k}{(M_k^2 - M_i^2)^2 + M_i^2\Gamma_k^2}$$

Resonant condition: $M_k - M_i \simeq \Gamma_k/2$

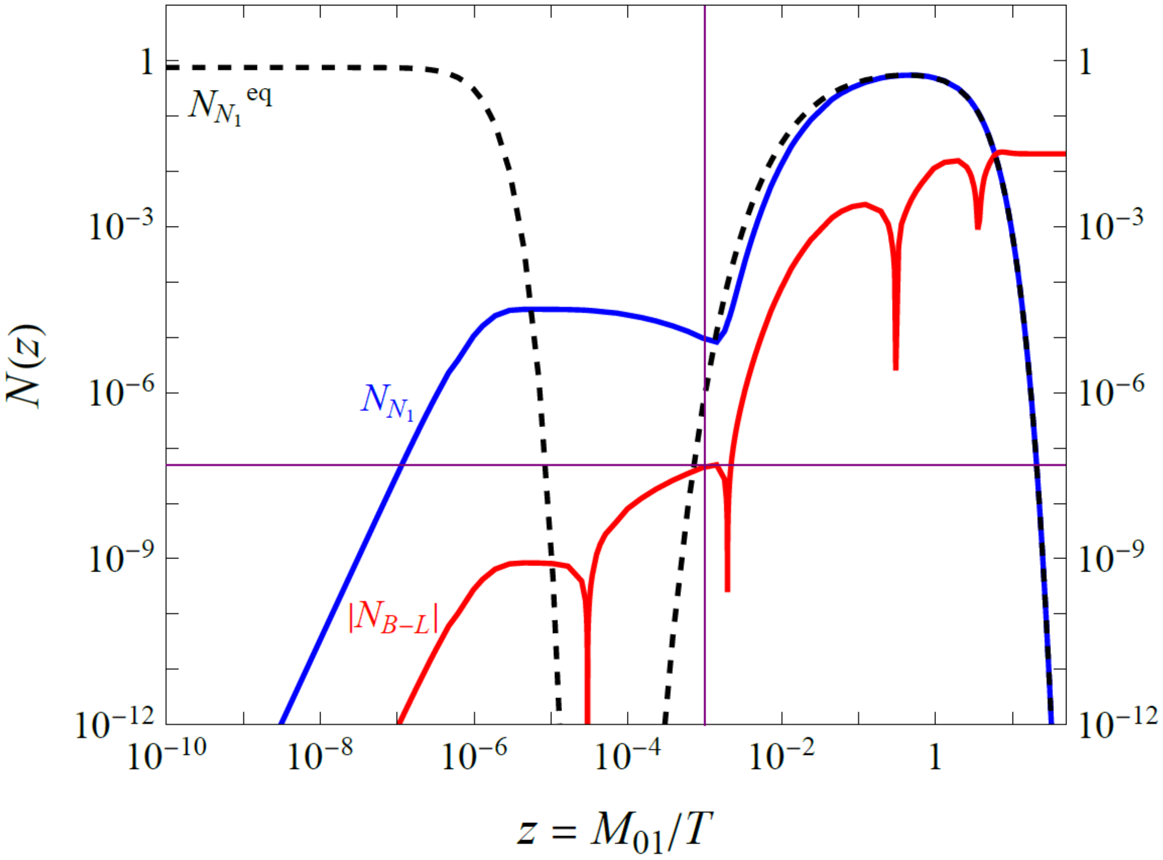
$$y = \sqrt{2}\hat{M}_N^{1/2} R \hat{M}_\nu^{1/2} U^\dagger_{\mathrm{PMNS}}/v$$

$$R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{\omega_1} & s_{\omega_1} \\ 0 & -s_{\omega_1} & c_{\omega_1} \end{pmatrix} \begin{pmatrix} c_{\omega_2} & 0 & s_{\omega_2} \\ 0 & 1 & 0 \\ -s_{\omega_2} & 0 & c_{\omega_2} \end{pmatrix} \begin{pmatrix} c_{\omega_3} & s_{\omega_3} & 0 \\ -s_{\omega_3} & c_{\omega_3} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

[K. Moffat et al (2018)]

Input parameters for Benchmark point

$$m_\phi = 10^{-2}\ \text{eV}\ (T_* \simeq 2.7\times 10^3\ \text{GeV})$$
$$M_{01} = 0.1\ \text{GeV}$$
$$M_{*1} = 2.4\times 10^5\ \text{GeV}$$
$$\Delta M_{12} \equiv M_{02} - M_{01} = 0.5\times 10^{-19}\ \text{GeV}$$
$$\Delta M_{13} \equiv M_{03} - M_{01} = 4.0\times 10^{-19}\ \text{GeV}$$
$$M_{*i}/M_{*1} = M_{0i}/M_{01}\ (i=2,3)$$
$$m_{\nu_1} = 0\ \text{eV}$$
$$\delta_{CP} = \pi,\ \alpha_1 = \alpha_2 = 0$$
$$\omega_1 = \omega_2 = 0,\ \omega_3 = 0.2e^{i\pi/4}$$



Photon dilution and Sphaleron factors

$$\eta_B = \frac{a_{\rm spa}}{f} N_{B-L}$$

$$a_{\rm spa} = 28/79, \ f = 1232/43$$

Photon dilution
from 1st LG to BBN

$$\eta_L = \frac{1}{f'} N_{B-L}$$

$$f' = 176/43$$

Photon dilution
from 2nd LG to BBN

$$f = g_S^\star / g_S^0$$

Dilution factor is given by the ratio between relativistic effective degrees of freedom in entropy, because the photon number increases by the annihilation process of the particles.

[W. Buchmuller et al (2002)]