Effect of Gravitational Cooling on Ultralight Dark Matter System Compared to Dynamical Friction

Hyeonmo Koo[1,2]

(Collaborated with Jae-Weon Lee[3], Dongsu Bak[1,2])

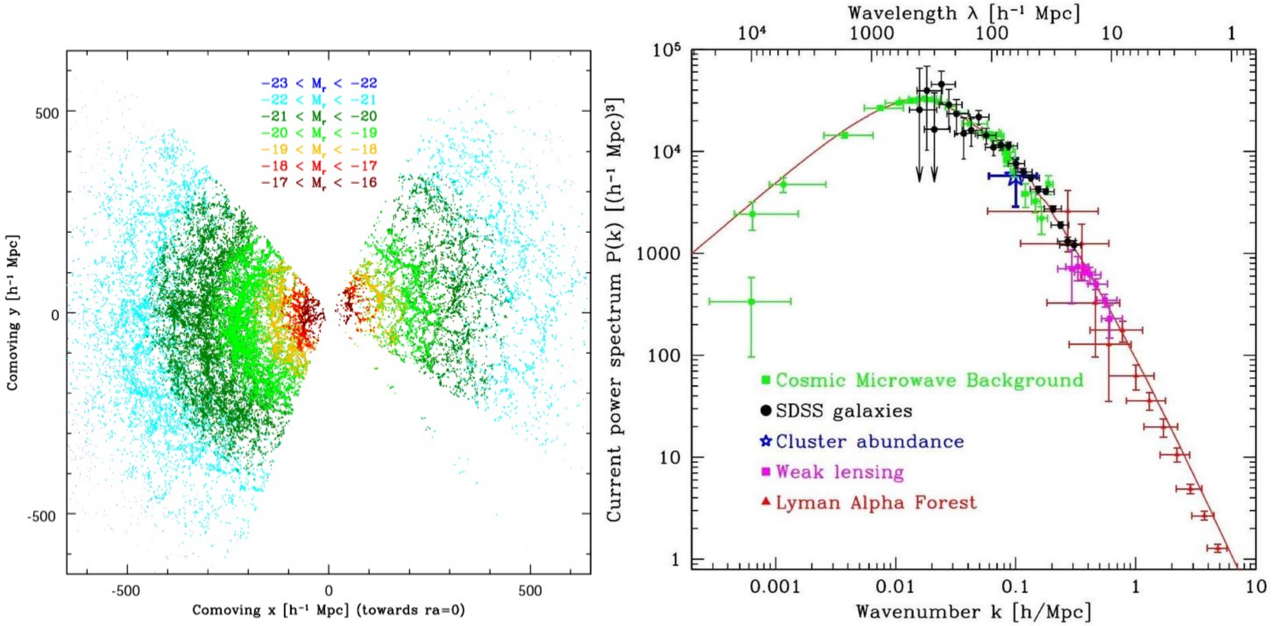
[1]Department of Physics, University of Seoul

[2]Natural Science Research Institute, University of Seoul

[3]Department of Electrical and Electronic Engineering, Jungwon University

Does ΛCDM explain our Universe well?

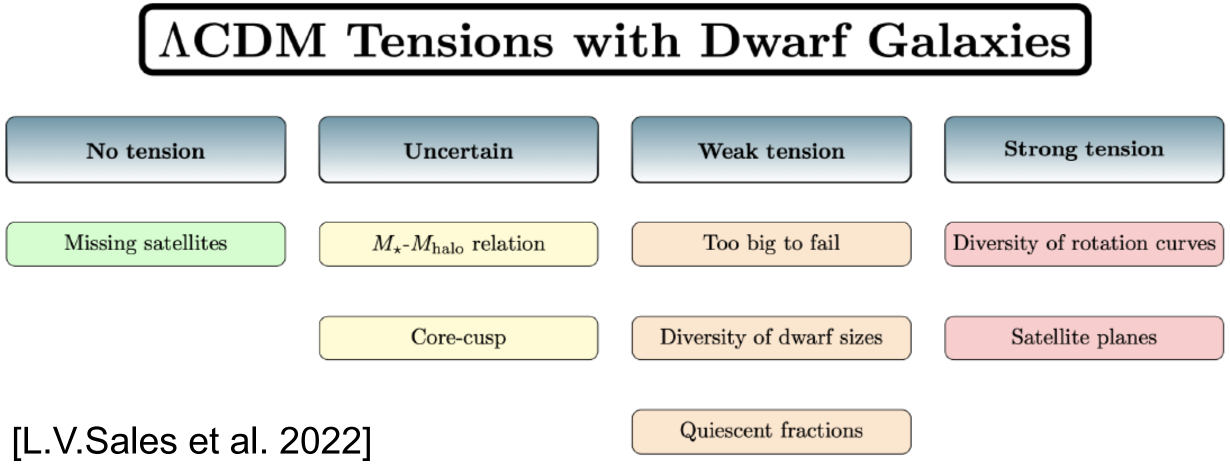
For Large Scale: ~O(Mpc)



[M.Tegmark et al. 2003]

- ΛCDM could explain the observed large-scale structure of our Universe well.

For Small Scale: ~O(kpc)

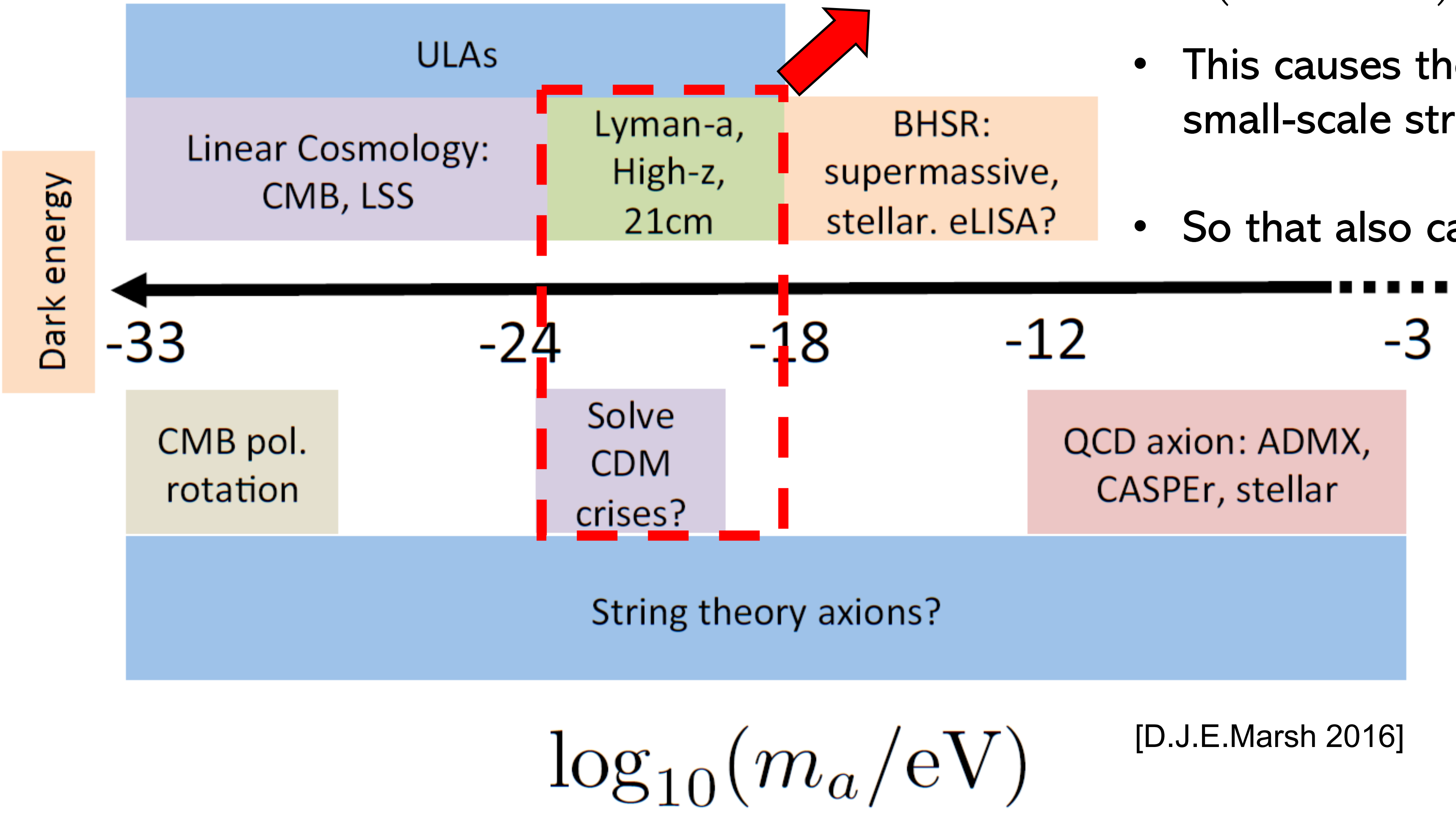


[L.V.Sales et al. 2022]

- Some problems can be solved by adding baryonic matter in ΛCDM simulation, but still not enough.
- Alternative Dark Matter model is required to solve!

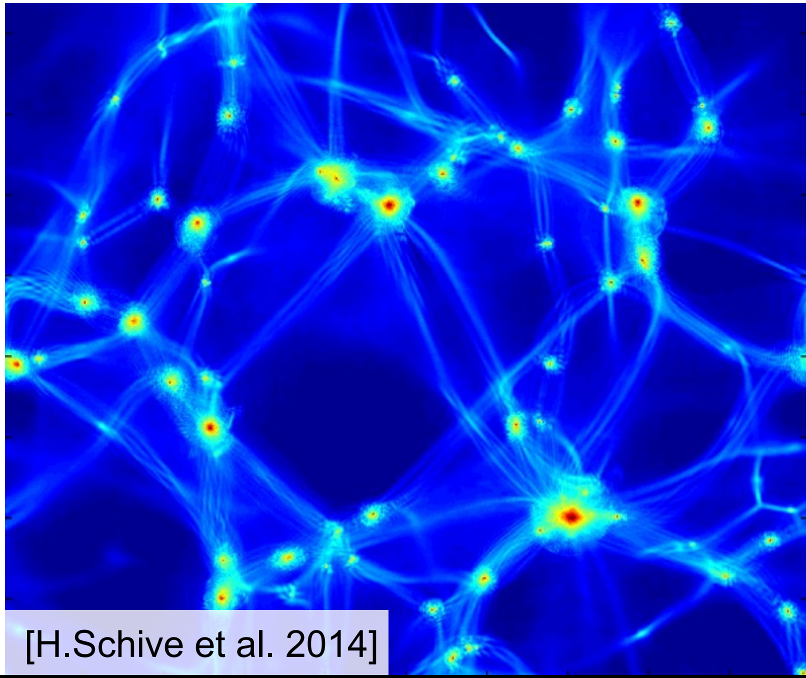
UltraLight Axion Dark Matter(ULDM)

$$\lambda_{\mathrm{dB}} = \frac{h}{m\sigma} \sim 1.204\mathrm{kpc}\left(\frac{10^{-22}\mathrm{eV}/c^2}{m}\right)\left(\frac{100\mathrm{km/s}}{\sigma}\right)$$



[D.J.E.Marsh 2016]

- This causes the wave nature, suppressing the small-scale structure formation in our universe.
- So that also called “Fuzzy Dark Matter(FDM)”



[H.Schive et al. 2014]

$$\mathcal{L} = \frac{1}{2} g^{\mu\nu}(\partial_\mu \phi)(\partial_\nu \phi) - V(\phi), \qquad V(\phi) = m^2 f_a^2 \left(1 - \cos\frac{\phi}{f_a}\right) = \frac{1}{2} m^2 \phi^2 - \frac{\lambda}{4!}\phi^4 + \cdots$$

$$\lambda = \left(\frac{m}{f_a}\right)^2$$

- The relic density of ALP today is:

$$\Omega_{\text{axion}} \sim 0.1 \left(\frac{f_a}{10^{17}\text{GeV}}\right)^2 \left(\frac{m}{10^{-22}\text{eV}}\right)^{1/2}$$ [L.Hui et al. 2016]

All interaction terms of $\sim f_a^{-1}$ are negligible!

This simplifies ALP system to $V(\phi) \simeq \frac{1}{2} m^2 \phi^2$

Weak-Gravity Limit $ds^2 = \left(1 + \frac{2\Phi}{c^2}\right) c^2 dt^2 - \left(1 - \frac{2\Phi}{c^2}\right)\eta_{ij} dx^i dx^j$

Nonrelativistic Limit $\phi = \sqrt{\frac{\hbar^3 c}{2m}} \left(\psi e^{-i\frac{mc^2 t}{\hbar}} + \psi^* e^{i\frac{mc^2 t}{\hbar}}\right)$

[Schrodinger-Poisson Eq.]

$$-\frac{\hbar^2}{2m}\nabla^2\psi + m\Phi\psi = i\hbar\frac{\partial\psi}{\partial t}$$

$$\nabla^2\Phi = 4\pi G m|\psi|^2$$

Madelung Formalism of SP eq.

$$-\frac{\hbar^2}{2m}\nabla^2\psi + m\Phi\psi = i\hbar\frac{\partial\psi}{\partial t}$$

$\psi = \sqrt{\frac{\rho}{m}}e^{i\theta}$

$\vec{v} = \frac{\hbar}{m}\nabla\theta$

1. Continuity eq. $\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\vec{v}) = 0$

2. Euler eq. $\left(\frac{\partial}{\partial t} + \vec{v}\cdot\nabla\right)\vec{v} = -\nabla\Phi - \nabla Q$

3. Poisson eq. $\nabla^2\Phi = 4\pi G\rho$

- Q roles the quantum pressure, which resists to the gravitational collapse of ULDM system.

$$Q = -\frac{\hbar^2}{2m^2}\left(\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}\right)$$

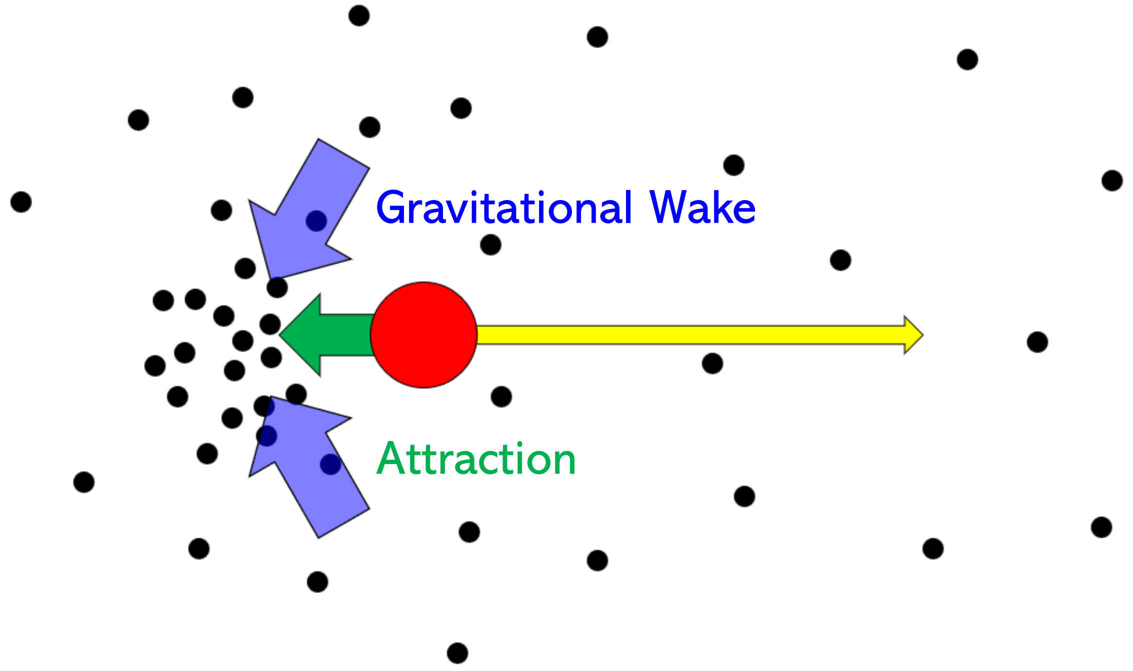
Dynamical Friction of CDM/ULDM

- An object of mass M traveling inside a particle field of density $\bar{\rho}$ with velocity v experiences the dynamical friction as below:

$$F_{\mathrm{DF}} = 4\pi\bar{\rho}\left(\frac{GM}{v}\right)^2 C_{\mathrm{DF}}$$

DF Coefficient!

1. For Classical Particles (CDM)



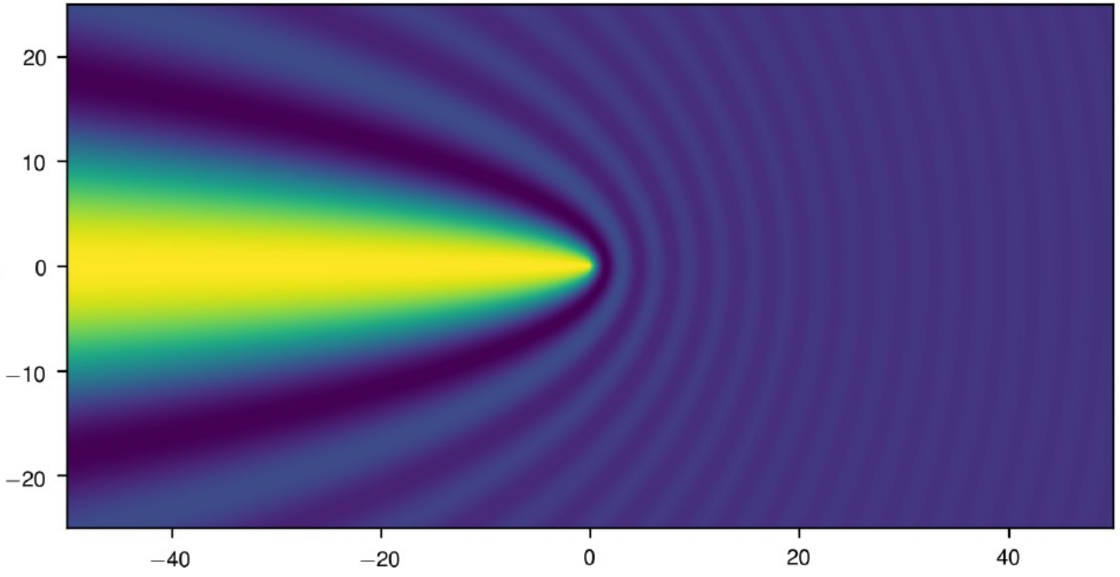
- The object loses its kinetic energy by gravitational interactions with surrounding field particles. [Chandrasekhar 1943]

- For sufficiently large v…

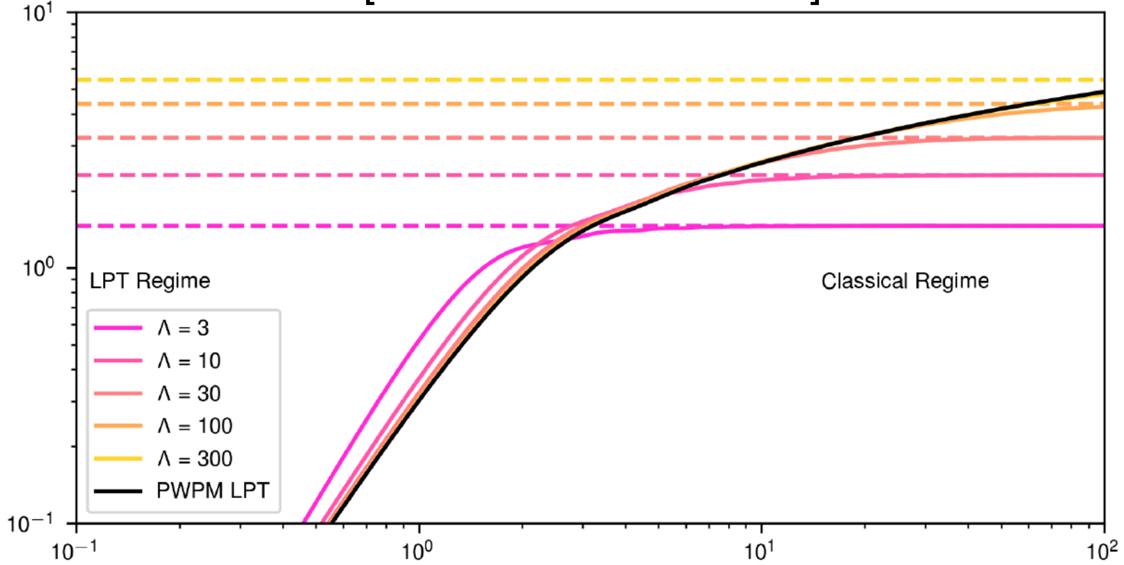
$$\frac{d\vec{v}_{\mathrm{M}}}{dt} = -4\pi G^2 M m_a n \log\Lambda \frac{\vec{v}}{v^3}$$ [Binney & Tremaine]

$$C_{\mathrm{DF}} = \log\Lambda = \log\frac{b_{\max}}{\max(r_{\mathrm{h}}, GM/v^2)}$$

2. For Ultralight Dark Matter



[L.Lancaster et al. 2019]



- Linear Perturbation of Madelung eq. explains the dynamical friction of ULDM well.

- C_{DF} also can be derived from exact solution of SP eq.

$$C_{\mathrm{DF}}(\tilde{b},\beta) \simeq \begin{cases} \frac{1}{3}\tilde{b}^2 \; (\tilde{b} \ll 1) \\ \log(2\tilde{b}) - 1 + \mathrm{Re}\Psi(1+i\beta) \; (\tilde{b} \gg 1) \end{cases}$$

$$\tilde{b} \equiv \frac{mvb}{\hbar} \qquad \beta = \frac{GMm}{\hbar v}$$

[L.Hui et al. 2016]

- For $\tilde{b} \gg 1$, the dynamical friction of ULDM becomes similar to that of CDM(classical limit).

- Dynamical friction of ULDM is generally less than of CDM.

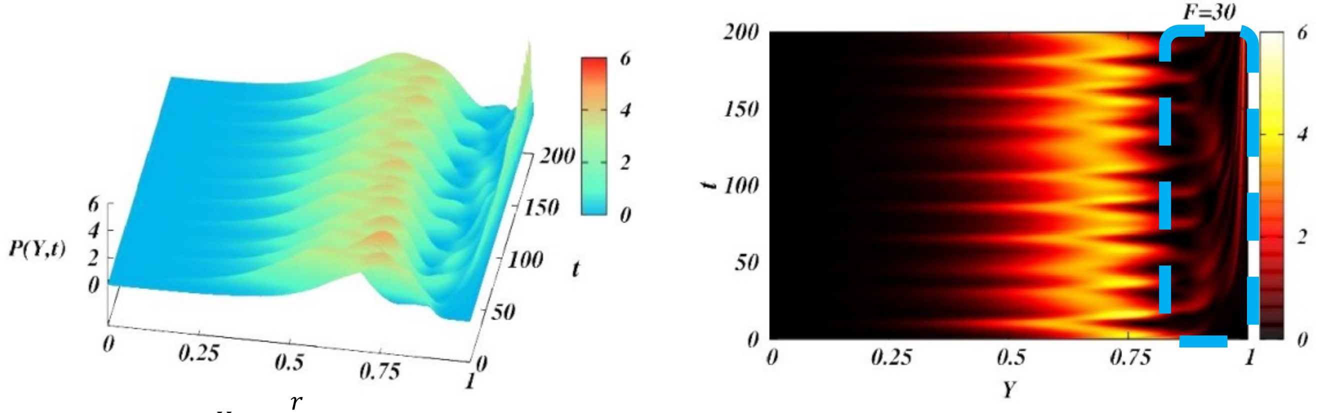
Gravitational Cooling Effect of ULDM

What is Gravitational Cooling Effect of ULDM?

- A relaxation process of the unstable configuration, through the emission of DM particles. The wave nature allows high-k modes with high energy $\epsilon_k = \frac{\hbar^2 k^2}{2m}$ to escape from the potential well.

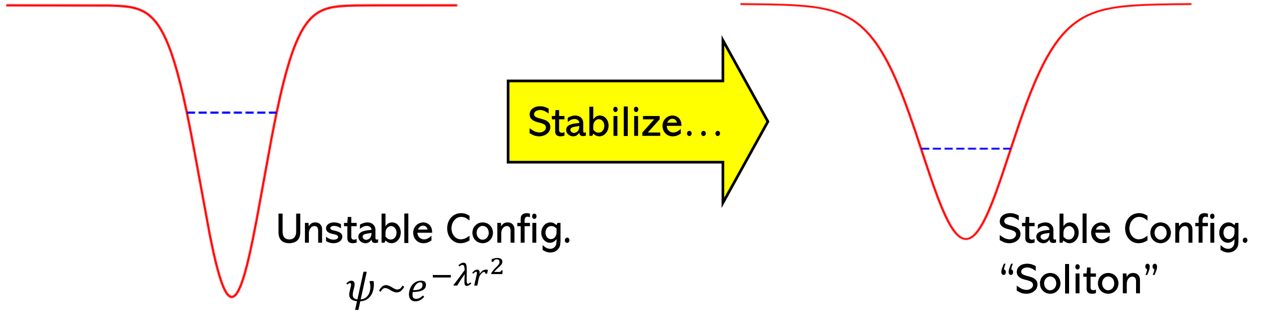
- Its timescale $\tau_{\rm gc}$ has a relation with total mass of system as:

$$\tau_{\rm gc} = \left(\frac{\hbar}{m}\right)^3 \frac{1}{(GM_{\rm tot})^2}$$
$$= 0.372\text{Myr}\left(\frac{10^{-22}\text{eV}}{m}\right)^3 \left(\frac{10^9 M_\odot}{M_{\rm tot}}\right)^2$$



$Y = \frac{r}{1+r}$ [Dongsu Bak et al. 2018]

- This effect had only been derived from numerical calculation yet.



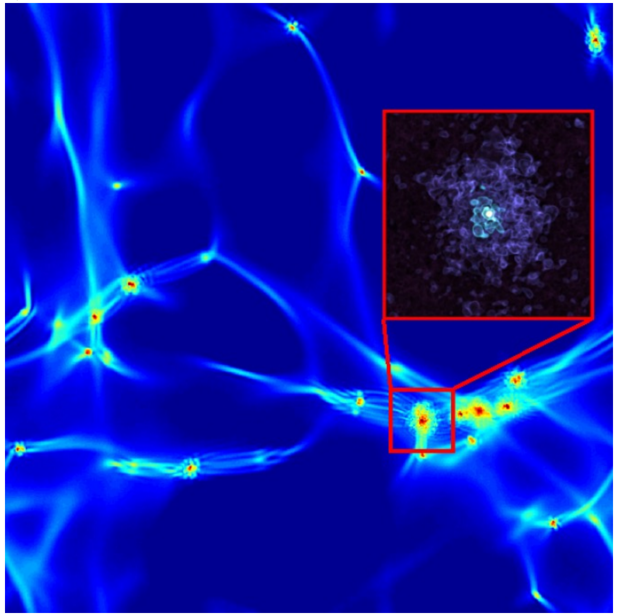
Solitonic Core and ULDM Halo

- The most stable ground-state solution of SP eq.

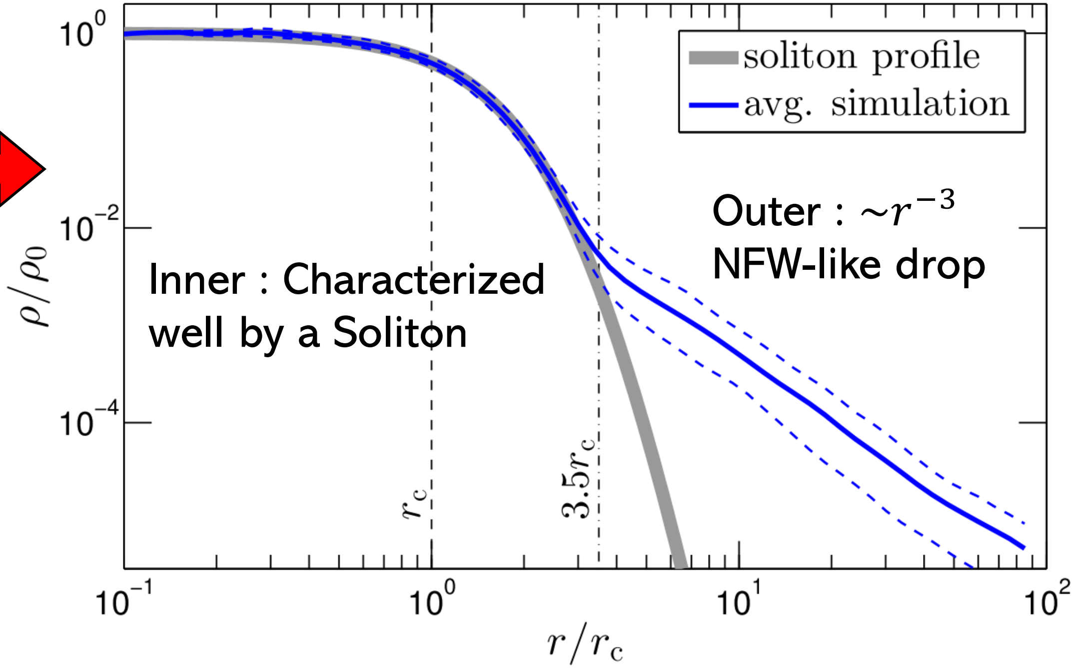
$$\begin{cases} r_{1/2} \cong 0.335\text{kpc}\dfrac{10^9 M_\odot}{M}\left(\dfrac{10^{-22}\text{eV}}{m}\right)^2 \\ \rho_c \cong 7.05 M_\odot/\text{pc}^3 \left(\dfrac{m}{10^{-22}\text{eV}}\right)^6 \left(\dfrac{M}{10^9 M_\odot}\right)^4 \end{cases}$$

$$\rho_{\text{soliton}}(r) \cong \frac{\rho_c}{\{1 + 0.091(r/r_c)^2\}^8}$$ [L.Hui et al. 2016]

$r_h \cong 1.452 r_c$



[P.Mocz et al. 2017]



Some Results of Numerical Simulations

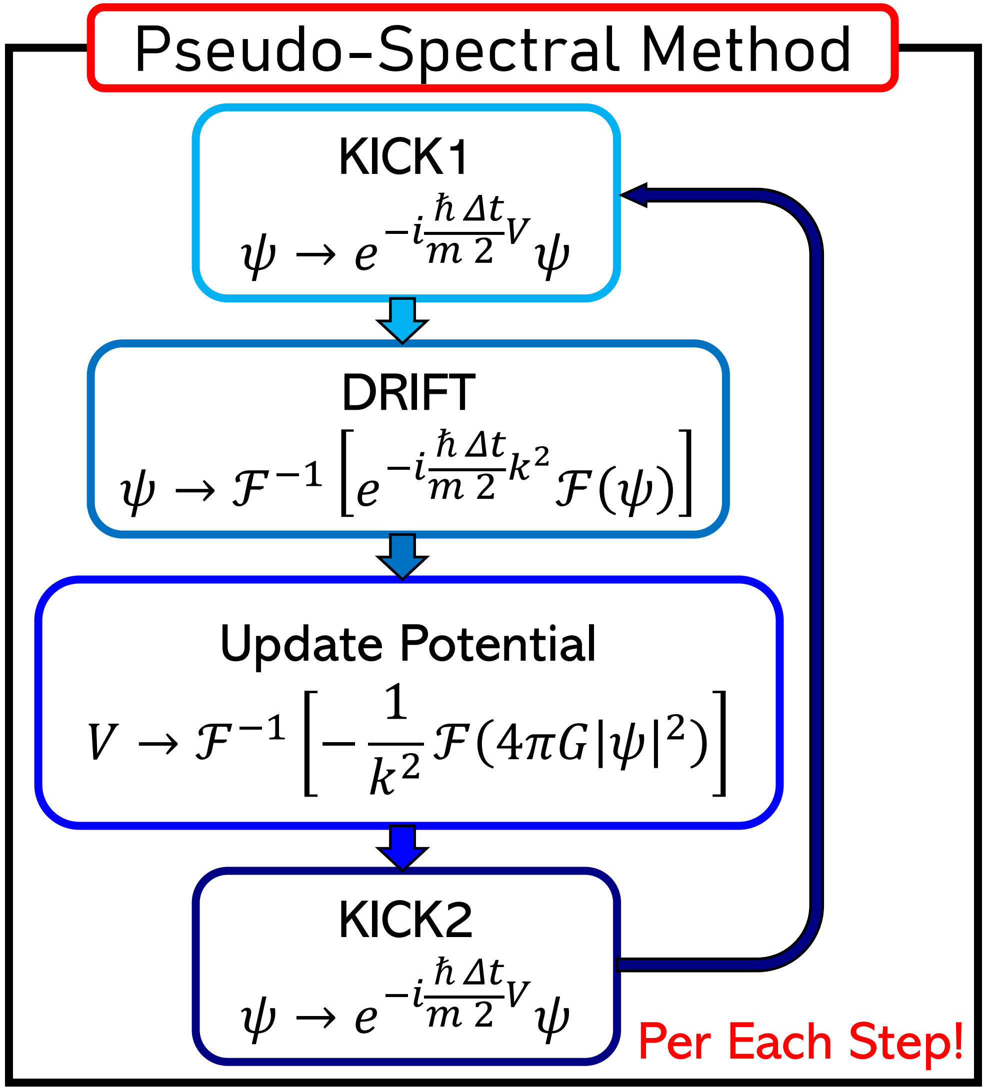
Schrodinger-Poisson eq. Solver

- PyUltraLight : Code for solving SP equation using Pseudospectral Method with periodic boundary condition [F.Edwards et.al. 2018]
- PyUltraLight2 : SP equation coupled with N-body particles solved by 4th order Runge-Kutta method [W.Wang et.al. 2021]

$$\begin{cases} -\frac{\hbar^2}{2m}\nabla^2\psi + m(\Phi_U + \Phi_N)\psi = i\hbar\frac{\partial\psi}{\partial t} \\ \nabla^2\Phi_U = 4\pi G|\psi|^2 \end{cases} \oplus \begin{cases} \Phi_N(\vec{x}) = -\sum_j \frac{GM_j}{|\vec{x}-\vec{x}_j|} \\ \frac{d^2\vec{x}_j}{dt^2} = -\nabla_j \sum_{k\neq j} \frac{GM_k}{|\vec{x}_j - \vec{x}_k|} - \nabla\Phi_U(\vec{x}_j) \end{cases}$$

- Timestep is chosen to prevent the artifact from being canceled out by constructive interference of ψ when $\arg(\psi) \geq \pi$.

$$\Delta t = \frac{m}{\hbar}\frac{\Delta x^2}{\pi}$$

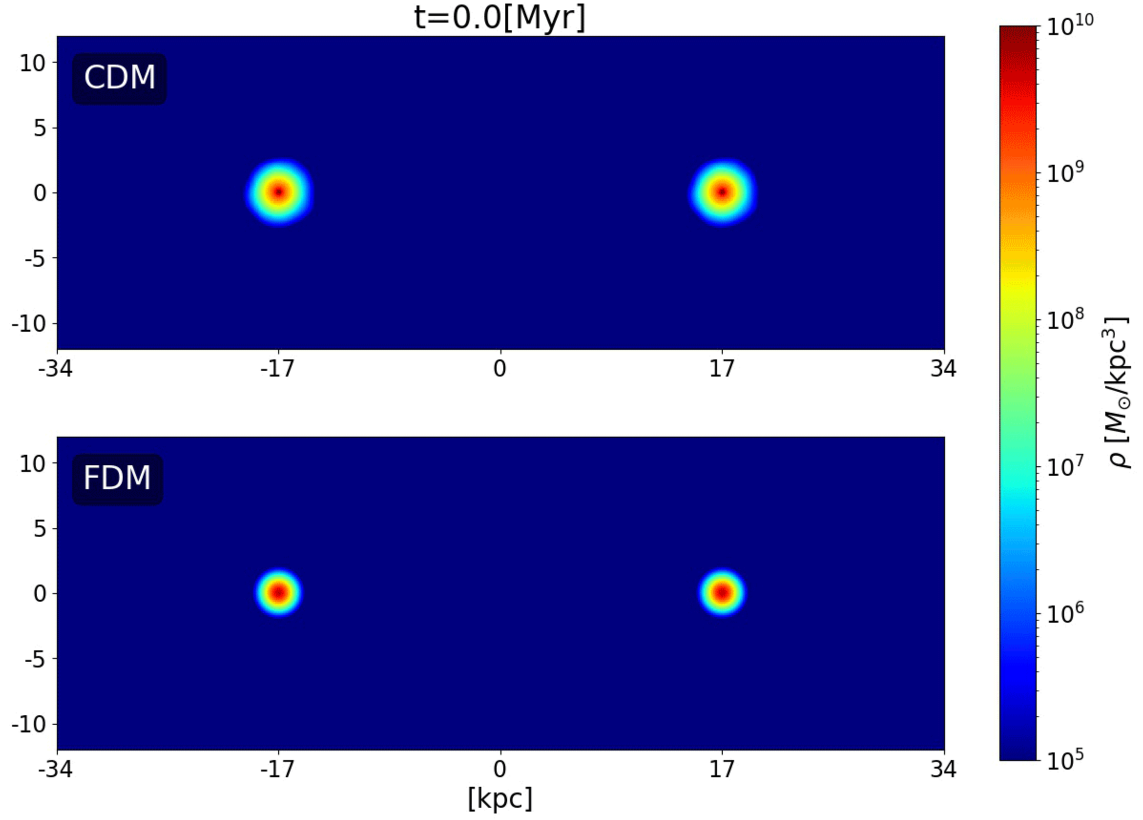


1. Head-on Collision of same-mass Halos

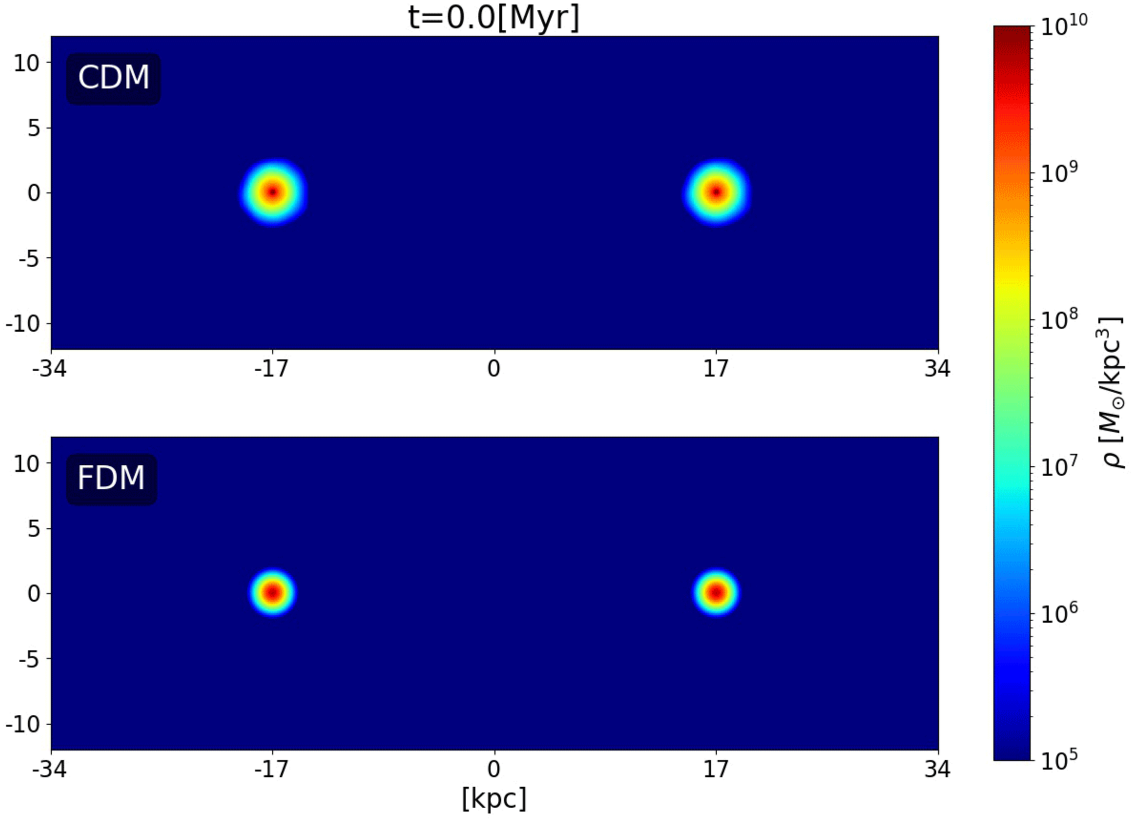
[H.Koo et al. 23XX.XXXXX]

- Each CDM/ULDM(FDM) halo has a mass of $M = 2\pi \times 10^8 M_\odot$ & ULDM particle mass $m = 10^{-22}\mathrm{eV}/c^2$

$v_0^{\mathrm{rel}} = 112.78\mathrm{km/s}$

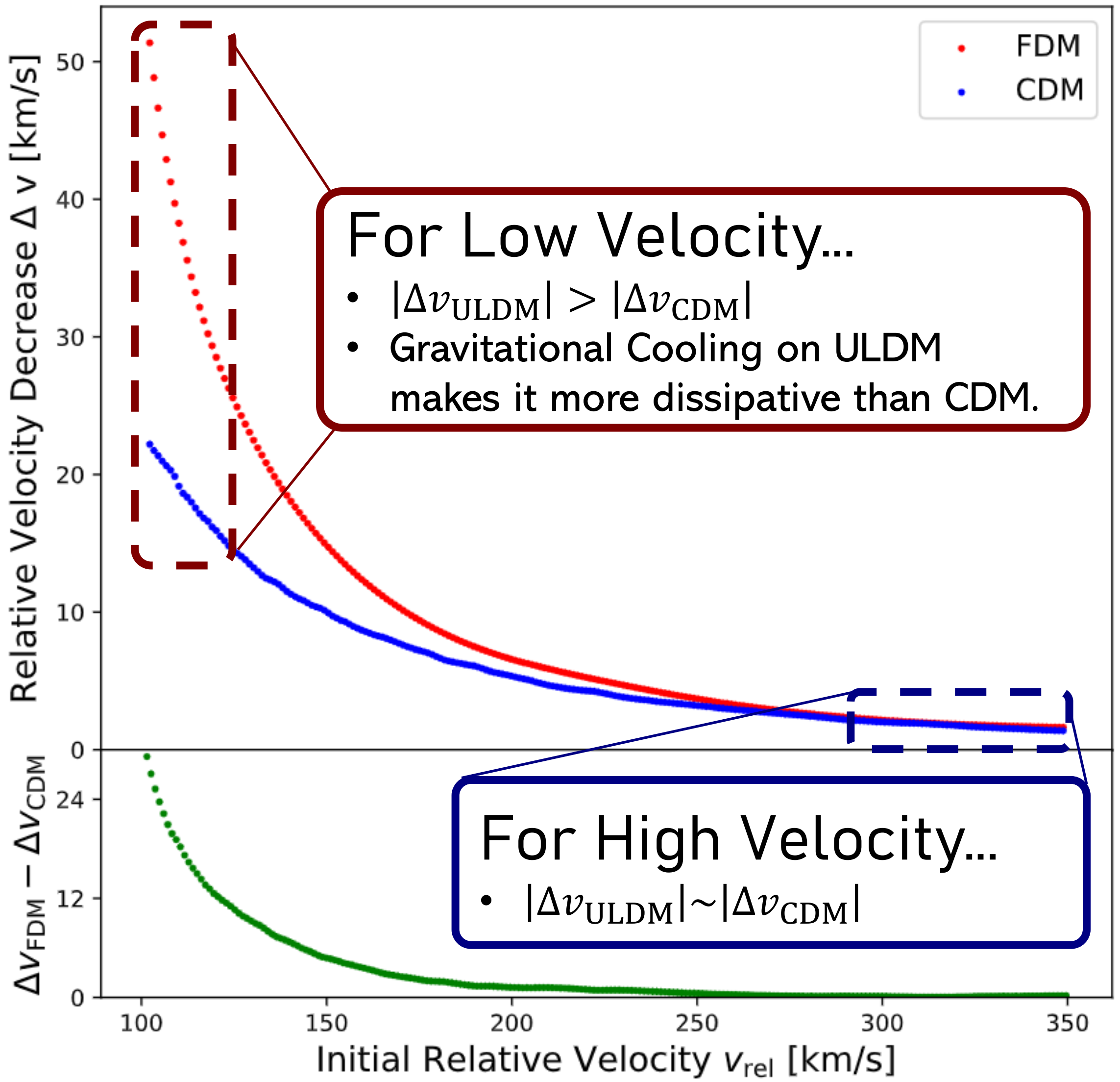


$v_0^{\mathrm{rel}} = 225.56\mathrm{km/s}$



1. Head-on Collision of same-mass Halos

[H.Koo et al. 23XX.XXXXX]



$$q \equiv \frac{\hbar}{m v r_{1/2}}$$

$v = 100\sim350\mathrm{km/s}$ → $q = 0.103\sim0.360$

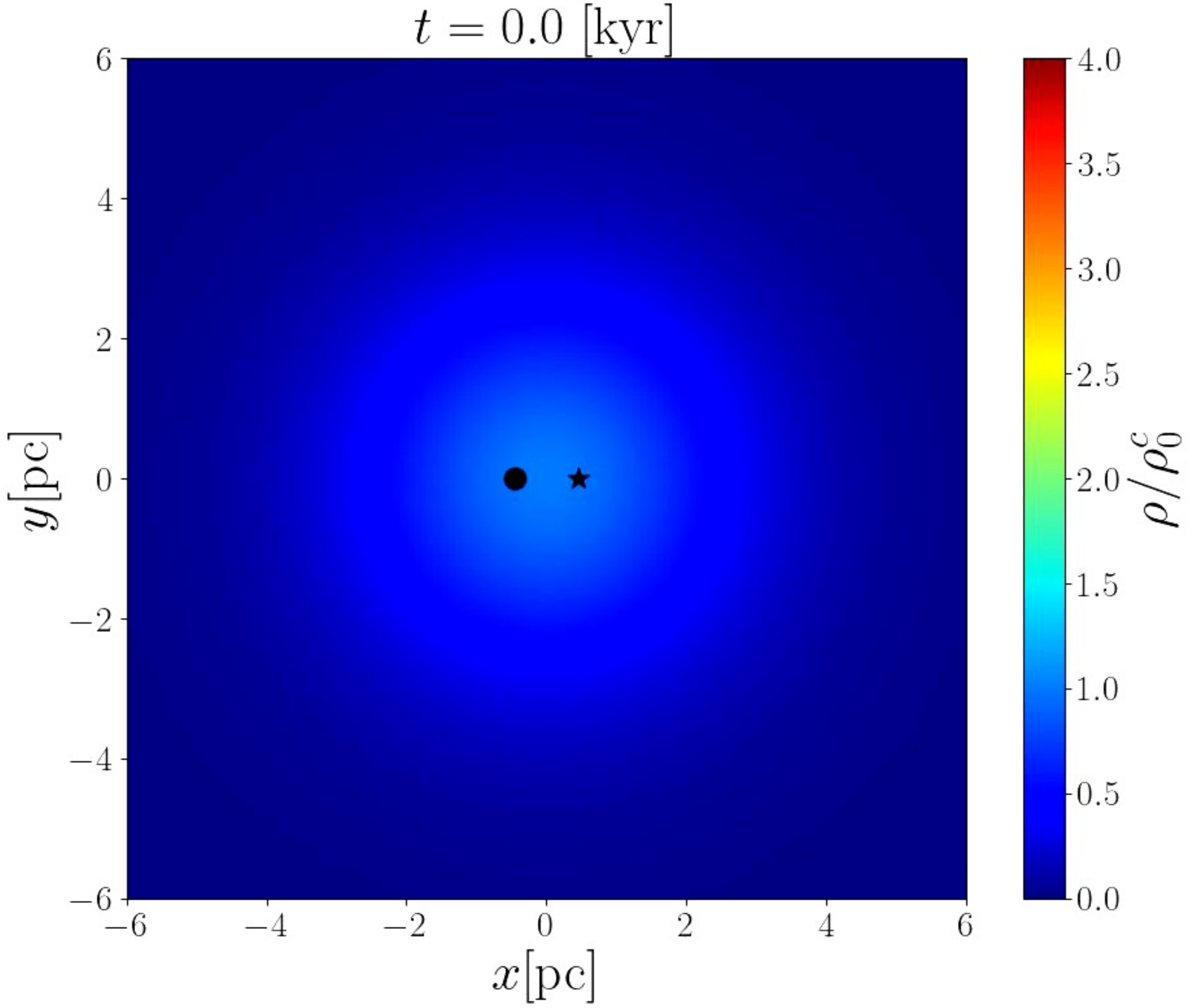
$$\left|\frac{\Delta v}{v}\right|_{\mathrm{CDM}} = A_c q^2(-\log q + B_c) \begin{cases} A_c = 15.93 \pm 0.126 \\ B_c = -0.169 \pm 0.008 \end{cases}$$

$$\left|\frac{\Delta v}{v}\right|_{\mathrm{ULDM}} = A_u q^3(1 + B_u q^2) \begin{cases} A_u = 3.692 \pm 0.041 \\ B_u = 15.63 \pm 0.294 \end{cases}$$

Fitting Function Derived from [D.Bak et al. 2010.14738]

2. Supermassive BH Binary in ULDM Halo

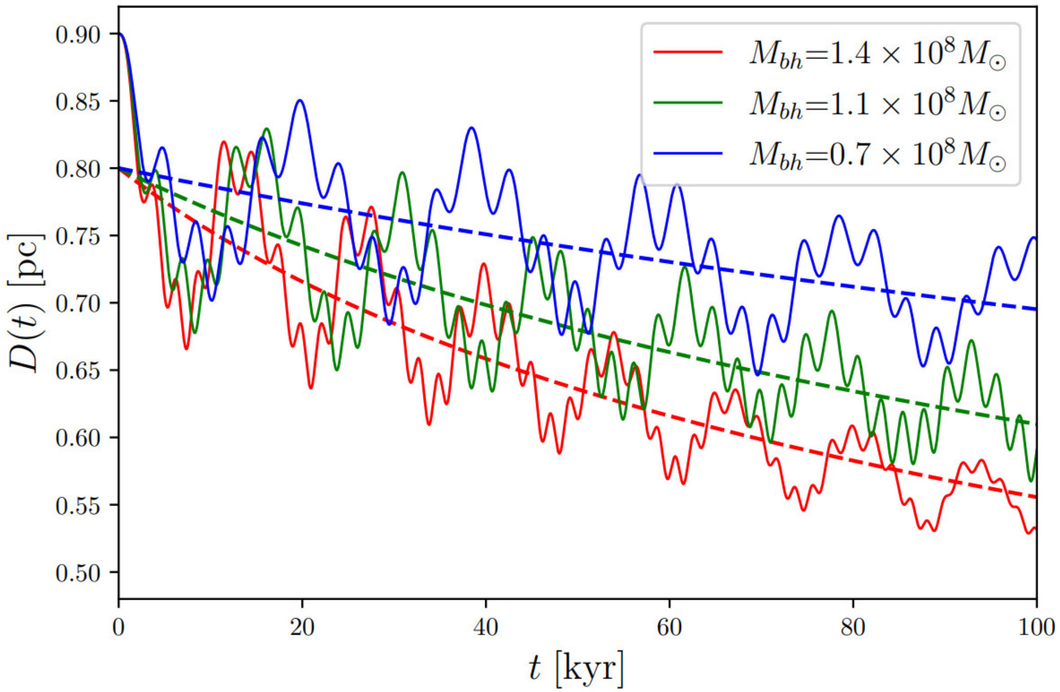
[H.Koo et al. 2311.03412]



$m = 10^{-21}\mathrm{eV}/c^2$
$M_\mathrm{s} = 10^9 M_\odot$
$M_\mathrm{bh} = 10^8 M_\odot$

2. Supermassive BH Binary in ULDM Halo

[H.Koo et al. 2311.03412]



Gravitational Cooling on total system could be

1. Global Mode : Breathing soliton with period τ

$$\tau = \tau_0 \left(\frac{10^{-21}\mathrm{eV}}{m}\right)^3 \left(\frac{10^9 M_\odot}{M_s}\right)^2 \left(1 + 2\gamma_g \frac{M_{\mathrm{bh}}}{M_s}\right)^{-2}$$

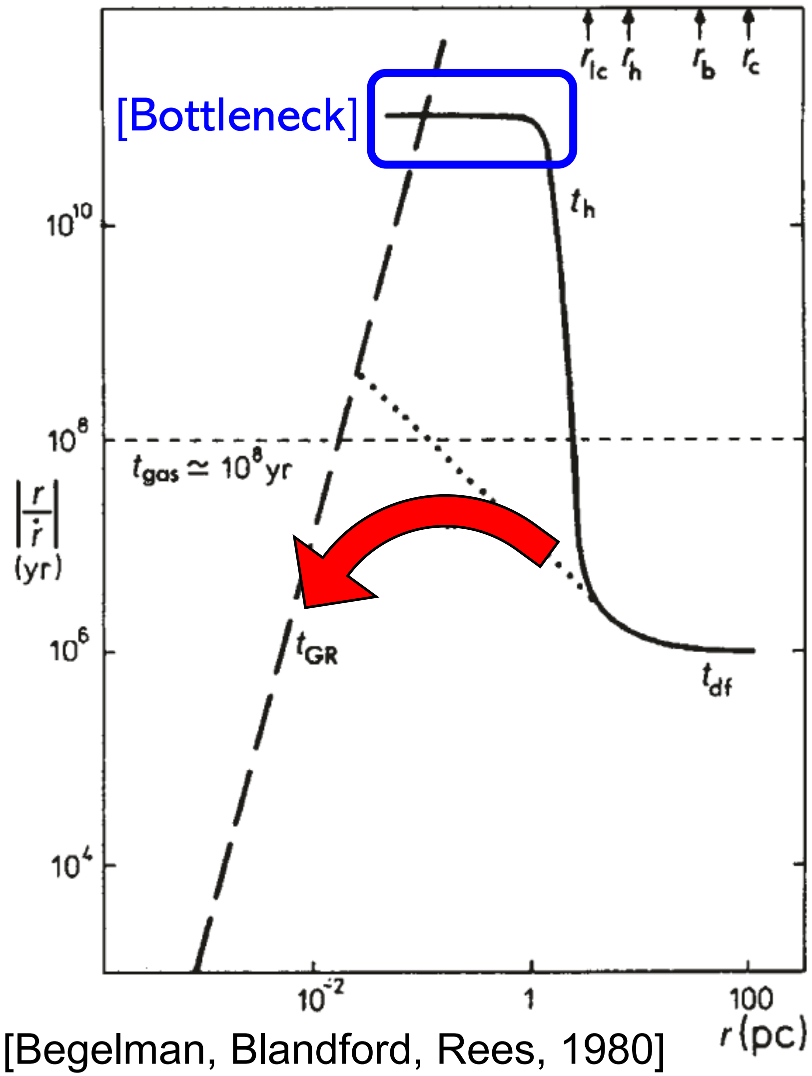
$$\begin{cases} \gamma_g = 1.974 \pm 0.013 \cong 2 \\ \tau_0 = (32.11 \pm 0.115)\mathrm{kyr} \end{cases}$$

2. Local Mode : Orbital Decay of SMBHB

$$\bar{D}(t) = \bar{D}_0 (1 + 3Q\bar{D}_0^3 t)^{-\frac{1}{3}}$$

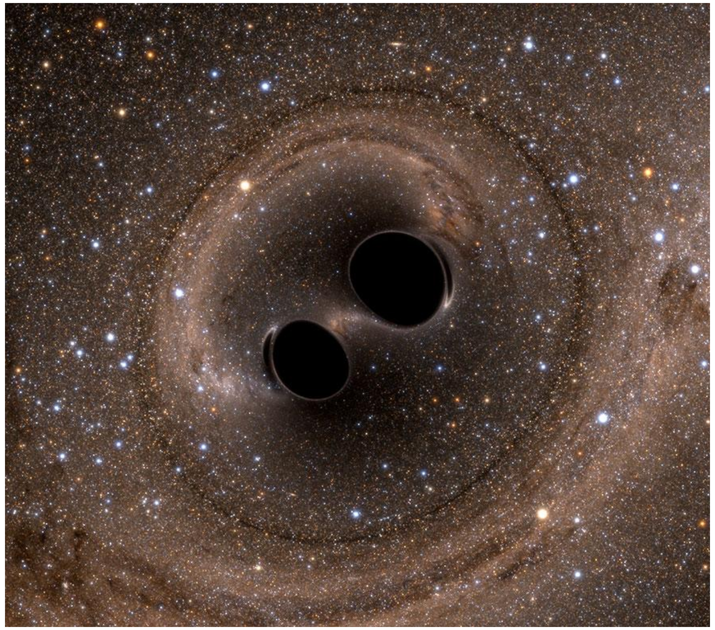
$$Q = \kappa \left(\frac{m}{10^{-21}\mathrm{eV}/c^2}\right)^9 \frac{(M_s + 2\gamma_l M_{\mathrm{bh}})^6}{M_{\mathrm{bh}}} \quad \begin{cases} \kappa = \dfrac{29.58 \pm 1.39}{\left[\mathrm{kyr} \cdot \mathrm{pc}^3 \cdot \left(10^{10} M_\odot\right)^5\right]} \\ \gamma_l = 1.476 \pm 0.067 \cong 1.5 \end{cases}$$

Can ULDM solve the Final Parsec Problem?



[Begelman, Blandford, Rees, 1980]

- Final Parsec Problem : Numerical simulations with ΛCDM show that SMBHBs typically stall out at $10^{-2}\text{pc} < r < 1\text{pc}$ and take longer than the age of the universe to reach the era of gravitational radiation.



- For $10^5 M_\odot \leq M_{\rm bh} \leq 10^7 M_\odot$ inside ULDM soliton, the Gravitational Cooling Effect gives a hint for SMBHs to survive from the timescale-bottleneck!

Summary

- Ultralight Axion Dark Matter of $m \sim 10^{-22}\mathrm{eV}/c^2$ and $\lambda_{\mathrm{dB}} \sim O(\mathrm{kpc})$ could solve some problems that ΛCDM encounters.
- The Gravitational Cooling Effect of ULDM system is a sort of relaxation through the emission of DM particles from potential well. This is significant for ULDM halo dynamics, rather than the dynamical friction.
- The Gravitational Cooling also could give a hint to solve Final Parsec Problem.
- Adding self interaction $\mathcal{L}_{\mathrm{I}} \sim \lambda\phi^4$ is expected to give more precise constraint!

Appendix 1 : C_{DF} for ULDM

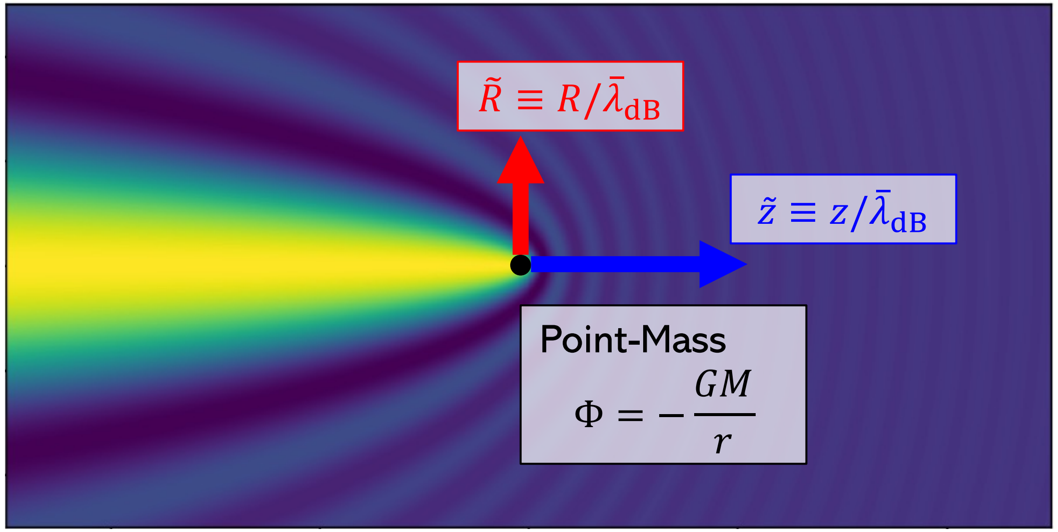
1. From Linear Perturbation

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0 \\ \left(\dfrac{\partial}{\partial t} + \vec{v} \cdot \nabla\right) \vec{v} = -\nabla \Phi - \nabla Q \\ \nabla^2 \Phi = 4\pi G \rho \end{cases}$$

$$\rho = \bar{\rho}(1+\alpha)$$
$$\vec{v} = v_0 \hat{z} + \delta \vec{v}$$
$$\psi_0(\vec{x}) = \sqrt{\bar{\rho}} e^{i\frac{m v_0 z}{\hbar}}$$

$$\left(\frac{\partial}{\partial t} - v_0 \frac{\partial}{\partial z}\right)^2 \alpha - \nabla^2 \Phi + \frac{\hbar^2}{4m^2} \nabla^4 \alpha = 0$$

$$\alpha(\tilde{R}, \tilde{z}) = \frac{4\beta}{\pi} \int_0^\infty d\tilde{k}_R \int_{-\infty}^\infty d\tilde{k}_z \frac{\tilde{k}_R J_0(\tilde{k}_R \tilde{R}) e^{i \tilde{k}_z \tilde{z}}}{\tilde{k}^4 - 4\tilde{k}_z^2}$$



- Dynamical friction from overdensity $F_{\mathrm{DF}} = \bar{\rho} \int d^3\vec{x}\, \alpha(\vec{x}) (\hat{x}_\parallel \cdot \nabla)\, \Phi(\vec{x})$ leads to C_{DF} as:

$$C_{\mathrm{DF}}(\tilde{b}) = -\int_0^{\tilde{b}} d\tilde{R} \int_0^{\sqrt{\tilde{b}^2 - \tilde{R}^2}} d\tilde{z} \int_0^2 du \frac{\tilde{R} \tilde{z} J_0\left(\sqrt{2u - u^2}\, \tilde{R}\right)}{\left(\tilde{R}^2 + \tilde{z}^2\right)^{\frac{3}{2}}} \frac{\sin \tilde{z} u}{u}$$

Appendix 1 : C_{DF} for ULDM

2. From direct solution of SP eq. with Point-Mass

$$\psi(\tilde{R},\tilde{z}) = \sqrt{\bar{\rho}}e^{i\tilde{z}+\frac{\pi}{2}\beta}|\Gamma(1-i\beta)|\times M\left[i\beta, 1; i\left(\sqrt{\tilde{R}^2+\tilde{z}^2}+\tilde{z}\right)\right]$$

Where M is the confluent hypergeometric function.

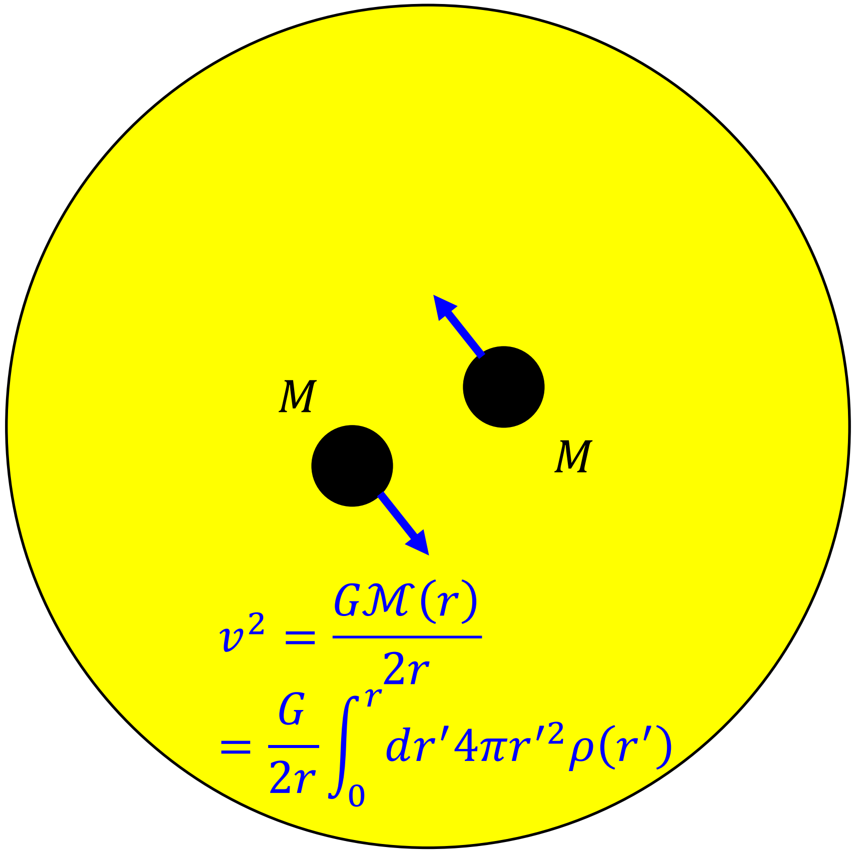
$$\alpha(\vec{x}) \equiv \frac{\rho(\vec{x})}{\bar{\rho}} - 1 = \frac{|\psi|^2}{\bar{\rho}} - 1$$

We samely put α into $F_{\mathrm{DF}} = \bar{\rho}\int d^3\vec{x}\alpha(\vec{x})(\hat{x}_{\parallel}\cdot\nabla)\,\Phi(\vec{x})$, then could get:

$$C_{\mathrm{DF}}(\tilde{b},\beta) = \frac{1}{2\beta}e^{\pi\beta}|\Gamma(1-i\beta)|^2\int_0^{2\tilde{b}} d\tilde{q}|M(i\beta,1;i\tilde{q})|^2\left(\frac{\tilde{q}}{\tilde{b}} - 2 - \log\frac{\tilde{q}}{2\tilde{b}}\right)$$

$$\simeq \begin{cases} \frac{1}{3}\tilde{b}^2\ (\tilde{b} \ll 1) \\ \log(2\tilde{b}) - 1 + \mathrm{Re}\Psi(1+i\beta)\ (\tilde{b} \gg 1) \end{cases}$$

Appendix 2 : Decay Timescale inside ULDM Halo

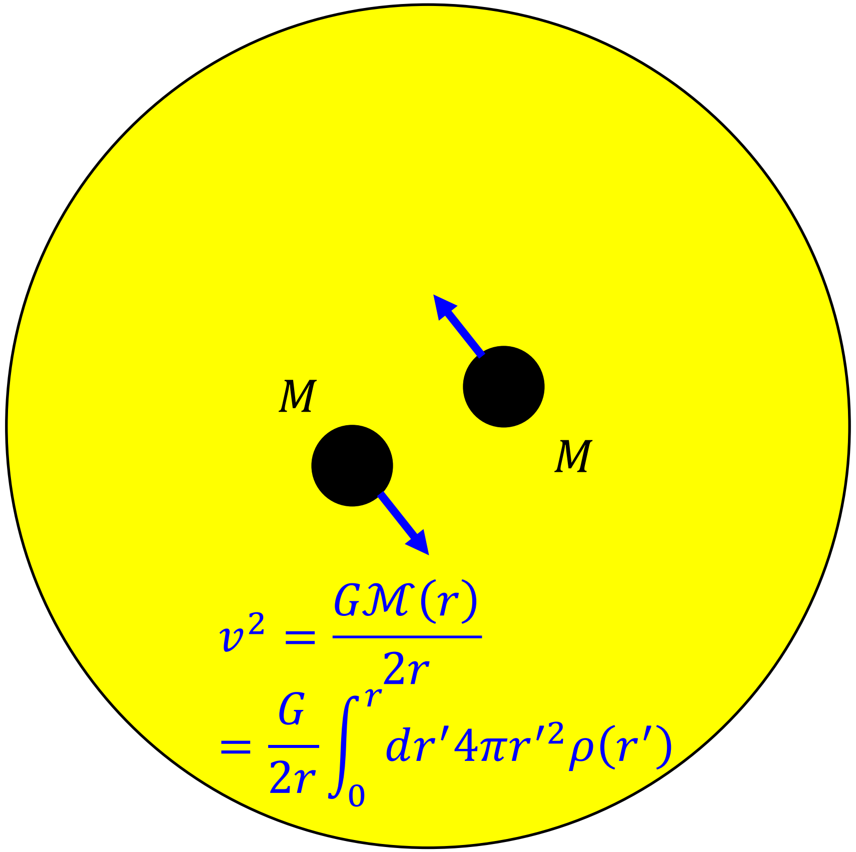


1. By Dynamical Friction

$$\tau = \frac{L}{r\times|F_{\mathrm{DF}}|} = \frac{Mvr}{r\times 4\pi\rho(r)(GM/v)^2 C(\tilde{r})}$$

$$= \frac{Mv^3}{4\pi\rho(r)(GM)^2\times\frac{1}{3}\left(\frac{mvr}{\hbar}\right)^2} = \frac{3\hbar^2\sqrt{\mathcal{M}(r)}}{8\pi m^2\rho(r)M\sqrt{2G^3r^5}}$$

Appendix 2 : Decay Timescale inside ULDM Halo

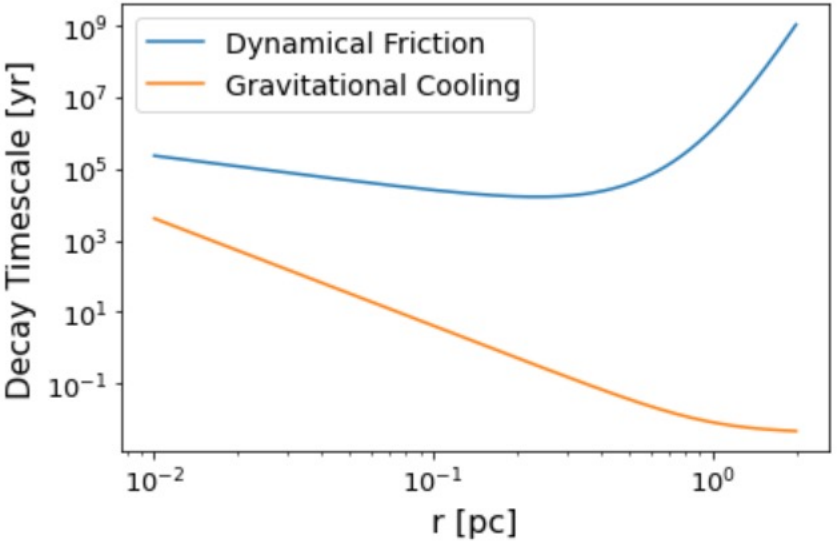


2. By Gravitational Cooling

$$D(t) = D_0(1 + 3QD_0^3 t)^{-\frac{1}{3}} \Rightarrow \tau = \left(\frac{1}{D}\frac{dD}{dt}\right)^{-1} = \frac{1}{QD^3}$$

Where $Q = \kappa\left(\frac{m}{10^{-21}\mathrm{eV}/c^2}\right)^9 \frac{(M_s + 2\gamma_l M_{bh})^6}{M_{bh}}$

$$\begin{cases} \kappa = \frac{29.58 \pm 1.39}{\left[\mathrm{kyr}\cdot\mathrm{pc}^3\cdot(10^{10}M_\odot)^5\right.} \\ \gamma_l = 1.476 \pm 0.067 \cong 1.5 \end{cases}$$



$m = 10^{-21}\mathrm{eV}c^2$
$M_s = 10^{10} M_\odot$
$M_{bh} = 10^7 M_\odot$